Network Filters And Transmission Lines

(NFTL)

Electronics IVth Sem IInd yr.

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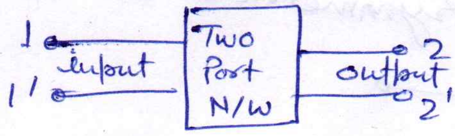
Electronics IVth sem Networks

* A network or circuit is an interconnection of a number of electrical components like resistors, inductors, capacitors, diodes, transeistors, transformer and energy sources. A network may have two or more terminals for making connections.

* A pair of terminals at which a signal may enter or leave a network is called a port.

* A port may defined as any pair of terminals into which energy is supplied or from which energy is withdrawn.

* When network has two seperate pair of terminals, then resulting network is called forer terminal network or Two port network.



* Active network - A network having generators or energy source is called Active network.

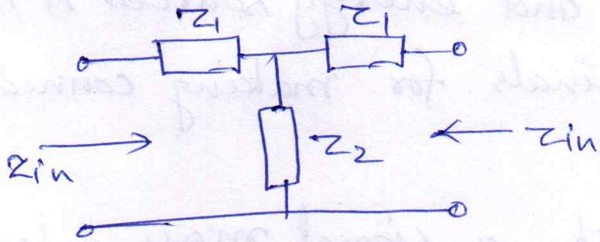
* Passive Network - A circuit element which does not generate an e.m.f. is called Passive elements.

* Symmetrical Network - whose electrical characteristics do not change when its input and output terminals are interchanged.

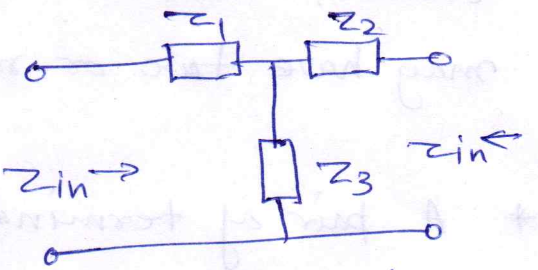
$$Z_1 Z_2 = Z_3$$

* Asymmeterical network - Whose electrical characteristics changes when its input and output terminals are interchanged.

$Z_1 Z_2 \neq Z_3$

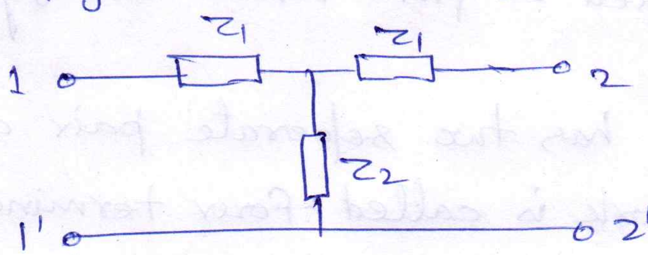


Symmetrical N/W

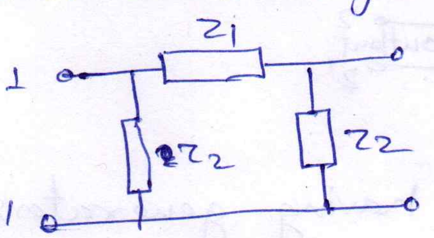


Asymmetrical N/W

* T- network - A N/W looks like 'T' is called the T section. T- section can be symmetrical or asymmetrical.



* π Section or π-Network - A network look like π is called π section. It can be symmetrical or Asymmetrical.



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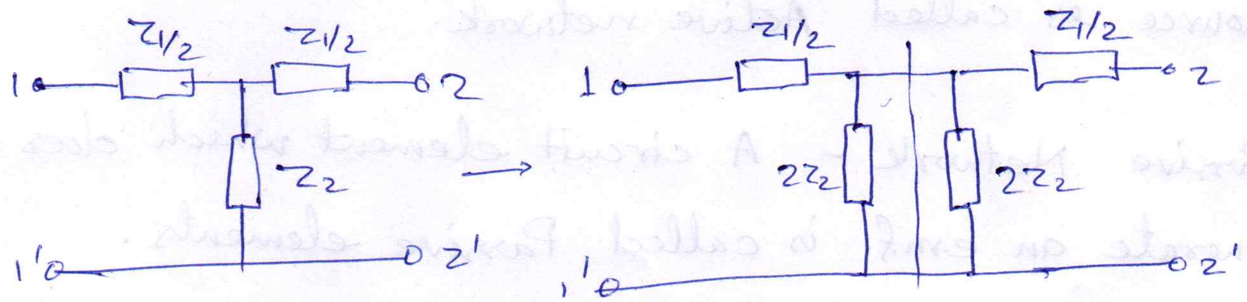
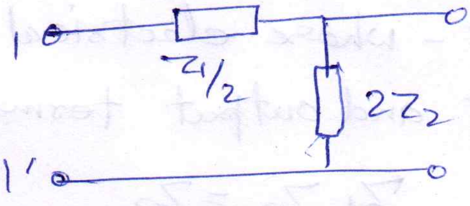


Half Section N/W



Topics -

Characteristics impedance, Propagation constant, Attenuation Constant, Phase shift constant, insertion loss.

① Characteristics impedance - (Z_0)

* Also known as surge impedance.
* Represented by Z_0
* Ratio of voltage & current amplitude of single wave propagating along line.
* Calculated for infinite line.
* SI unit of char^r impedance is Ω.
* If we remove first network in the fig. then the no. of networks remaining will still be infinite & hence the input impedance looking into the second n/w will be again Z_0.

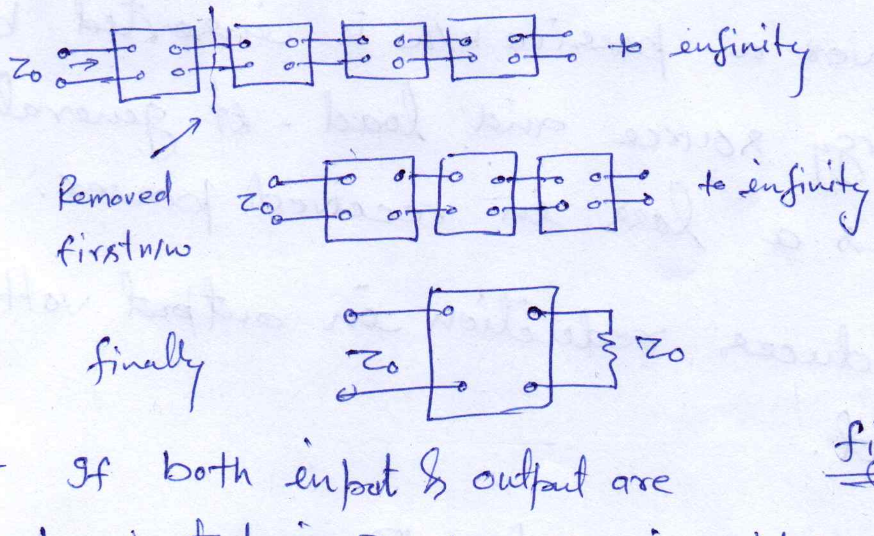


fig. 1

* If both input & output are terminated in Z_0 the n/w is said to be correctly or properly terminated.

(II). Propagation Constant -

* Thes is relation between input and output voltage and current.

* Represented by γ (gama)

$$\gamma = \alpha + j\beta$$

γ is propagation constant

α is attenuation constant unit is dB or nep/km.

β is phase constant unit is radian.

α is absalute magnitude and β is phase angle.

$$e^{\gamma} = e^{\alpha} \angle \beta$$

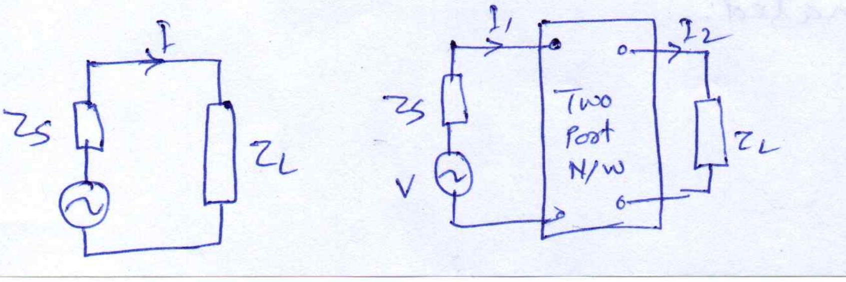
(III). Insertion loss -

* The no. of nepers or decibels by which the signal power is reduced by the insertion.

* whenever a passive n/w is inserted b/w an energy source and load - it generally produces a loss in received power.

* introduces reduction in output voltage & current.

* Measured in nep or dB.



Insertion loss

$$\text{in dB} = 20 \log_{10} \left| \frac{I_1}{I_2} \right| \text{ dB}$$

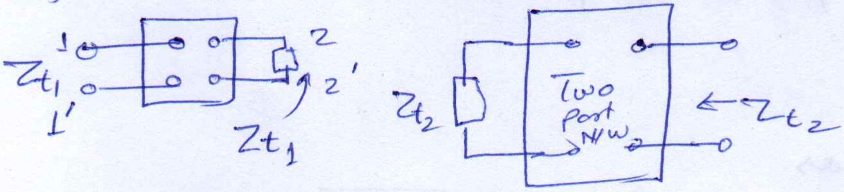
$$\text{in Nep} = \log_e \left| \frac{I_1}{I_2} \right| \text{ nep.}$$

Topics -

Iterative impedance, image impedance, image transfer constant, (for Asymmetrical network)

1. Iterative impedance -

* the input impedance measured at one pair of terminals when an infinite no. of such n/w are joined in series.



* Iterative impedances are different for the two pair of terminals.

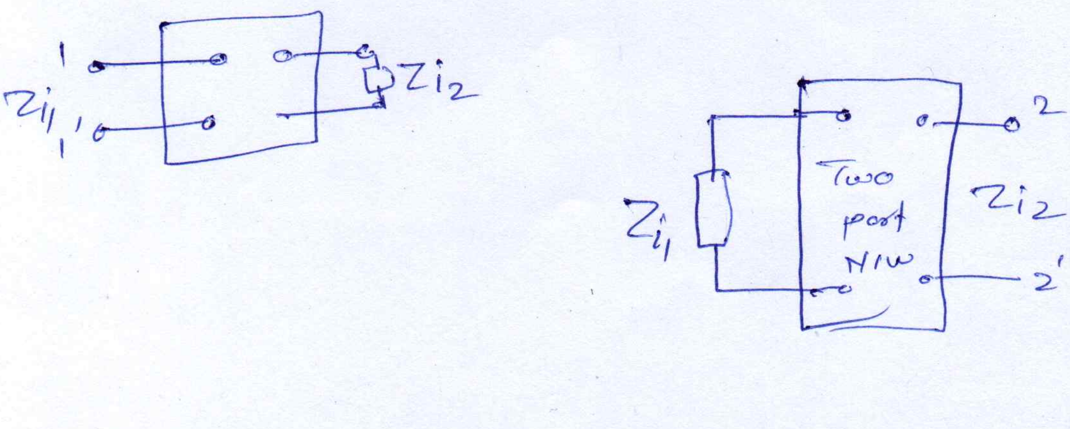
* These are termed as Z_{t_1} and Z_{t_2}.

2. Image Impedance

* If one impedance connected across the proper pair of terminals of the n/w.

* Denoted by Z_{i_1} and Z_{i_2}.

* Mirror effect presented by image impedances.



(iii) Image transfer Constant θ_i;

* The factors affecting propagation of energy is known as image transfer constant θ_i;

$$\theta_i = A_i + jB_i$$

A_i = Image attenuation constant in nep.

B_i = Image phase shift constant in radians.

* Represented as

$$e^{\theta_i} = \frac{V_1}{V_2}\sqrt{\frac{Z_{i_2}}{Z_{i_1}}}$$

(iv). Iterative transfer constant 'θ_t'

* $\theta_t = A_t + B_t$

A_t = Iterative attenuation constant in nep.

B_t = Iterative phase constant in radians.

* $$e^{\theta_t} = \frac{V_1}{V_2}$$

Topic - Attenuators.

Attenuator - An attenuator is a two port n/w used to reduce voltage current or power between its properly terminated input and output impedances in known amounts.

* Attenuation is independent of the frequencies.
* Attenuator are pure resistive networks.
* May be fixed or variable types.
* May be symmetrical & Asymmetrical.
* Attenuator circuit with constant attenuation is Called Pad.
* Attenuator are exactly opposite of amplifiers.
* Attenuator decreases the signal level.
* $$\text{Attenuation} = \frac{\text{Input Signal}}{\text{Output Signal}}$$
* Unit of attenuation is dB or nep.

$$\text{in dB} = 10 \log_{10}\left(\frac{P_1}{P_2}\right)$$

P_1 = input power P_2 = output power.

* $$\text{Attenuation in dB} = 20 \log_{10} N = 20 \log_e N \times \log_{10} e$$
$$= 20 \log_e N \times 0.434$$
$$= 8.686 \log_e N$$

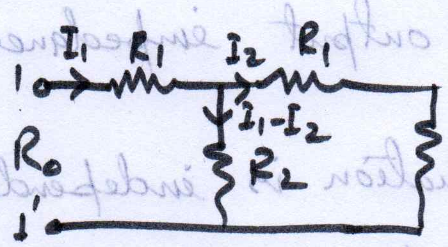
$\log_e N$ = Attenuation in nep.

* Attenuation in Nep = 0.1151 × Attenuation in dB

(11) Symmetrical Attenuators -

* Four types of attenuators
  1- T- attenuator 2 - π Attenuator 3 - Lattice Att.
  ~~4~~ Bridge T-Attunator.

1. T- Attenuator



Applying KCl in n/w.

$$I_2(R_1+R_0) - (I_1 - I_2) \cdot R_2 = 0 \quad \text{or} \quad I_2 R_1 + I_2 \cdot R_0 - I_1 R_2 + I_2 R_2 = 0$$

$$I_2(R_1+R_2+R_0) - I_1 R_2 = 0 \quad \text{or} \quad I_2(R_2+R_1+R_0) = I_1 R_2$$

$$\frac{I_1}{I_2} = \frac{R_2+R_1+R_0}{R_2}$$

$$\left(\because N = \frac{I_1}{I_2}\right)$$

$$N = \frac{R_2+R_1+R_0}{R_2} \quad \text{—(1)}$$

R_0 from input side

$$R_0 = R_1 + \frac{(R_1+R_0)R_2}{R_2+R_1+R_0} \quad \text{—(11)}$$

substituting equ^n (1) in (11).

$$R_0 = R_1 + \frac{(R_1+R_0)}{N}$$

by cross multiplication.

$$NR_0 = NR_1 + R_1 + R_0$$

$$NR_0 - R_0 = NR_1 + R_1$$

$$R_0(N-1) = R_1(N+1)$$

$$\boxed{R_1 = R_0 \frac{N-1}{N+1}} \quad \text{—(111)}$$

from $N = \dfrac{R_2+R_1+R_0}{R_2}$

$$NR_2 = R_2 + R_1 + R_0$$

$$NR_2 - R_2 = R_1 + R_0$$

$$R_2(N-1) = R_1 + R_0$$

Put the value of R_1 from eq^n (111)

$$R_2(N-1) = R_0\left(\frac{N-1}{N+1}\right) + R_0$$

$$= R_0\left(\frac{N-1}{N+1} + 1\right)$$

$$= R_0\left(\frac{N-1+N+1}{N+1}\right)$$

$$= R_0\left(\frac{2N}{N+1}\right)$$

$$\boxed{R_2 = R_0\left(\frac{2N}{N^2-1}\right)}$$

① · Symmetrical π - Attenuator -

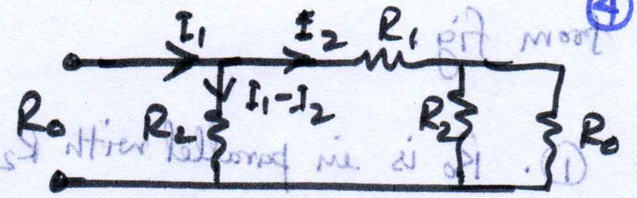


fig. 1

Applying KCL in N/w.

for convenience the circuit can be redrawn as shown in fig. 2

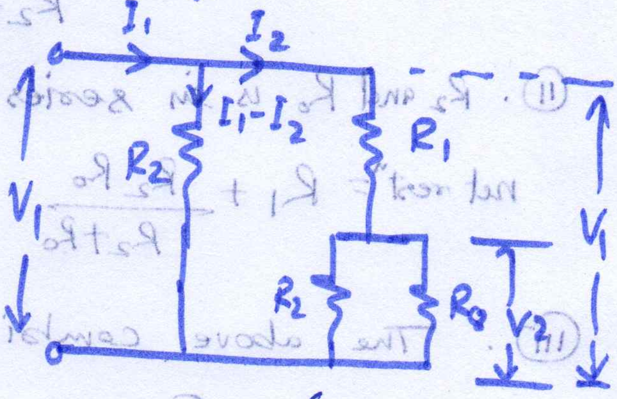


fig. 2

$$V_1 = (I_1 - I_2) R_2$$

or

$$V_1 = \left(R_1 + \frac{R_2 \cdot R_0}{R_2 + R_0} \right) I_2$$

by voltage division rule.

$$V_2 = \left(\frac{R_2 R_0}{R_2 + R_0} \right) I_2$$

$$N = \frac{V_1}{V_2} = \frac{\left(R_1 + \frac{R_2 \cdot R_0}{R_2 + R_0} \right) I_2}{\left(\frac{R_2 R_0}{R_2 + R_0} \right) I_2}$$

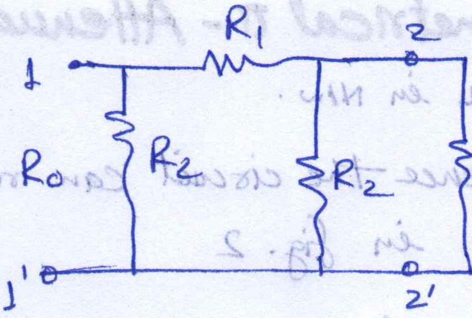
$$N = \frac{\frac{R_1 R_2 + R_1 R_0 + R_2 R_0}{\cancel{R_2 + R_0}}}{\frac{R_2 R_0}{\cancel{R_2 + R_0}}}$$

$$N = \frac{R_1 R_2 + R_1 R_0 + R_2 R_0}{R_2 R_0} = \frac{R_1 R_2}{R_2 R_0} + \frac{R_1 R_0}{R_2 R_0} + \frac{R_2 R_0}{R_2 R_0}$$ (by division)

$$\boxed{N = \frac{R_1}{R_0} + \frac{R_1}{R_2} + 1} \quad \text{——} \; ①$$

Now R_0 at input port 11'.

from fig:



(I). R_o is in parallel with R_2

$$\text{Net resistance} = \frac{R_2 R_o}{R_2 + R_o}$$

(II). R_2 and R_o is in series with R_1

$$\text{net rest}^n = R_1 + \frac{R_2 R_o}{R_2 + R_o}$$

(III). The above combination parallel with R_2

Therefore

$$R_o = \frac{R_2\left[R_1 + \frac{R_2 R_o}{R_2 + R_o}\right]}{R_2 + R_1 + \frac{R_2 R_o}{R_2 + R_o}} = \frac{R_2\left[\frac{R_1 R_2 + R_1 R_o + R_2 R_o}{R_2 + R_o}\right]}{R_2 + \left[R_1 + \frac{R_2 R_o}{R_2 + R_o}\right]}$$

$$R_o = \frac{R_2\left[\frac{R_1 R_2 + R_1 R_o + R_2 R_o}{R_2 + R_o}\right]}{R_2 + \left[\frac{R_1 R_2 + R_1 + R_o + R_2 \cdot R_o}{R_2 + R_o}\right]} \quad \text{----} \quad (ii)$$

$$\text{Put } N = \frac{R_1 R_2 + R_1 R_o + R_2 R_o}{R_2 R_o}$$

$$N R_2 \cdot R_o = R_1 R_2 + R_1 R_o + R_2 R_o$$

$$R_o = \frac{R_2\left(\frac{N R_2 R_o}{R_2 + R_o}\right)}{R_2 + \left(\frac{N R_2 \cdot R_o}{R_2 + R_o}\right)} = \frac{R_2\left(\frac{N R_2 R_o}{R_2 + R_o}\right)}{\frac{R_2(R_2 + R_o) + N R_2 R_o}{R_2 + R_o}}$$

$$R_o = \frac{R_2 (N R_2 \cdot R_o)}{R_2(R_2 + R_o + N R_o)}$$

$$R_o = \frac{N R_2 R_o}{R_2 + R_o + N R_o}$$

$$R_0 = \frac{N R_2 R_0}{R_2 + R_0 + N R_0}$$

By cross multiplying.

$$R_0 (R_2 + R_0 + N R_0) = N R_2 R_0 \quad \text{or} \quad R_2 + R_0 + N R_0 = N R_2$$

$$R_0 + N R_0 = N R_2 - R_2 \quad \text{or} \quad R_0 (1 + N) = R_2 (N-1)$$

$$\boxed{R_2 = R_0 \left(\frac{N+1}{N-1} \right)}$$

from equ^n

$$N = \frac{R_1}{R_0} + \frac{R_1}{R_2} + 1$$

$$N - 1 = \frac{R_1}{R_0} + \frac{R_1}{R_2} \quad \text{or} \quad N-1 = \frac{R_1 R_2 + R_1 R_0}{R_0 R_2}$$

$$(N-1) \cdot R_0 R_2 = R_1 R_2 + R_1 R_0$$

$$(N-1) \; R_0 R_2 = R_1 (R_2 + R_0)$$

Put R_2 value in above relation we get.

$$\boxed{R_1 = R_0 \left(\frac{N^2 - 1}{2N} \right)}$$

Q.1. Design a symmetrical T type attenuator to give 25 dB attenuation and to work into a characteristics impedance of 600 Ω

Sol. • Given data $R_o = 600\,\Omega$ $D = 25\,dB$

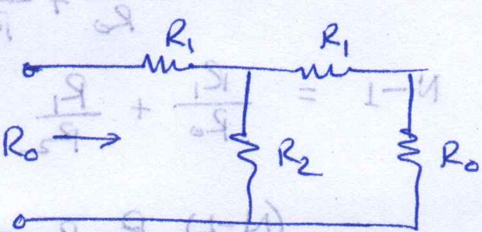
Design eqn =

$$R_1 = R_o\left(\frac{N-1}{N+1}\right) \qquad R_2 = R_o\left(\frac{2N}{N^2-1}\right)$$

$$N = \text{Antilog}\left(\frac{D}{20}\right)$$

$$N = \text{Antilog}\,\frac{25}{20} = \text{Antilog}\,1.25$$

$$N = 17.8$$



Now

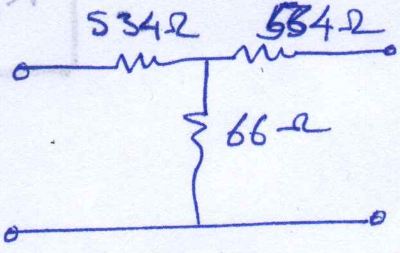
$$R_1 = R_o\left(\frac{N-1}{N+1}\right)$$

$$= 600\left(\frac{17.8-1}{17.8+1}\right)$$

$$= 600 \times (0.89)$$

$$= 534\,\Omega$$

Resulting circuit is



$$R_2 = R_o\left(\frac{2N}{N^2-1}\right)$$

$$= 600\left(\frac{2\times 17.8}{(17.8)^2-1}\right)$$

$$= 600\,(0.11) = 66\,\Omega$$

$R_1 = 534\,\Omega$ $\quad R_2 = 66\,\Omega$

Ans

Topic - Filters -

① Brief idea of filters. ② LPF ③ HPF ④ BPF ⑤ BSF

Filters - * Filters are frequency selective network that freely passes the desired band of frequencies while almost suppressing/attenuating all other bands.

Pass Band - A filter should produce no attenuation (0% att^n) in desired band known as Pass Band.

Stop Band/Attenuation Band - A filter should provide infinite or total attenuation at all other frequencies called stop band or attenuation band.

Cutoff frequency - The frequency which separates the pass band and attenuation band is known as cut-off frequency. denoted by 'f_c'.

* filter is constructed from purely reactive elements. (Combinations of Resistance, Capacitance, Inductance)

Applications -
* Communication Systems
* Telephone Circuits
* Instrumentations
* Telemetering Equipments
* T.V. Broadcasting
* Radio Broadcasting.
* Power Supplies.

Classification Of filters –

- Filter
  - Depending upon the Components Used
    - Active filter
    - Passive filters
  - Depending upon the relation b/w Series & Shunt impedance
    - Prototype or Constant K filters
    - m-derived filters
  - Depending upon the frequency characteristics
    - LPF
    - HPF
    - BPF
    - BSF

* Active filter

* Use OP-Amp and resistor or capacitor

* Passive filter –

* Use resistance Capacitance & Inductance.

* Prototype or Constant-K filter.

* Condition $Z_1 \cdot Z_2 = R_0^2 = K$

where R_0 = Real and constant. independent of frequ^n.

R_0 is called design impedance.

Drawback of constant k-filters –

(i). impedance matching becomes difficult.

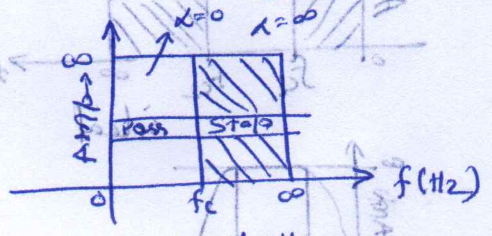
(ii). attenuation does not increase rapidly, rather it increase slowly in the pass band.

* m-derived filter

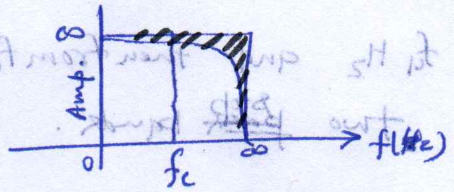
* In order to overcome the drawbacks of constant K filters are modified and named as m-derived filters.

1)- **Low Pass filter (LPF)** - A filter which allows all the frequencies upto the cutoff frequencies f_c to pass through it without attenuat-ion is called Low pass filter, (LPF)

An LPF attenuates all the frequencies greater than cut off frequency f_c.



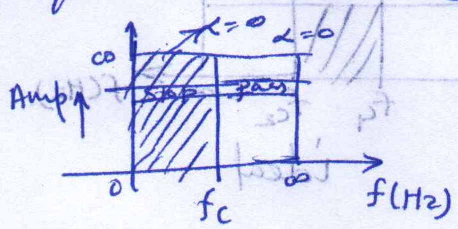
ideal filter



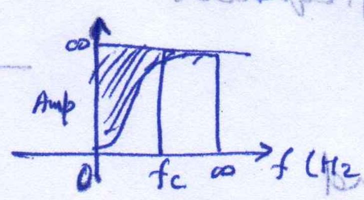
practical filter.

* In LPF pass band from 0Hz to f_c. So it passes all the freqn 0 to f_c.
* Stop band exist ~~from~~ above f_c.
* Small amount of attenuation is present.
* LPF uses for dc supply. voice frequency circuits upto 3kHz.
* They can be employed b/w tr and Rec. ant n prevent harmonics. (higher frequecies).

2)- **High Pass filter. (HPF)** - A filter which attenuates all the frequencies below a specified frequency f_c and passes all frequencies above f_c is called High Pass filter.



ideal



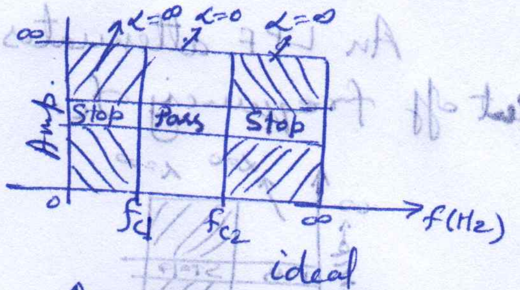
practical

* Stop band exist from 0Hz to f_c or pass band from f_c to onwards.
* opposite of LPF.
* Interchanging the components LPF it becomes HPF.
* Used in audio frequency circuits.

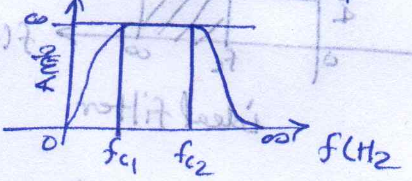
III) Band Pass filter - (BPF)

A filter which passes the signals between two specified cut-off frequencies and attenuates all other frequencies is called band pass filter.

* A BPF has one pass band two stop band and two cutoff frequ^n.



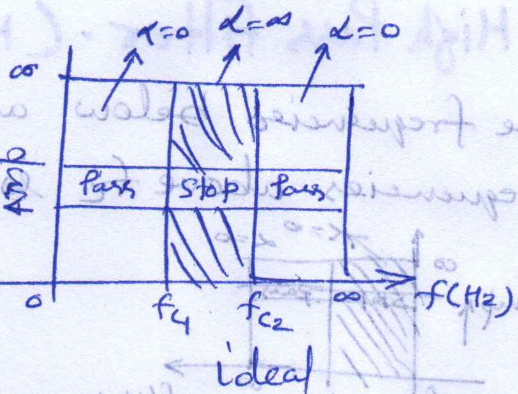
* 0 Hz to f_{c1} Hz and then from f_{c2} to onword two ~~pass~~ stop bands.



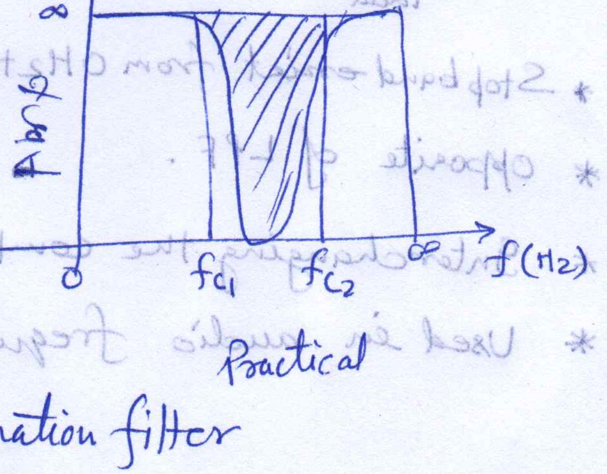
IV) Band Stop Filter - (BSF)

A filter which attenuates all the frequencies between two specified cut off frequencies f_1 & f_2 and passes all other frequencies is called Band stop filter.

* It is exactly opposite of BPF



* BSF attenuates the frequnces between f_1 and f_2.
* In BSF two pass band exist from 0 Hz to f_{c1} and from f_{c2} onwards.
* Stopband exists b/w f_{c1} and f_{c2} - so BSF reject the desired frequency range.



* BSF also known as <u>Band Elemination filter</u>

Filters –

① LPF –
LPF passes all frequencies without attenuation upto the cutoff frequency f_c and attenuates all other frequencies $> f_c$.

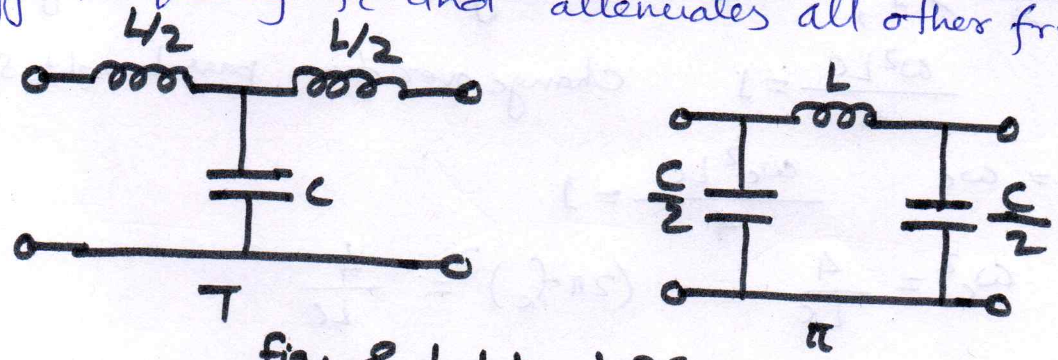


fig. Prototype LPF

for constant K protybe filter.

$$Z_1 Z_2 = R_o^2 = K$$

$$Z_1 = j\omega L \qquad Z_2 = \frac{1}{j\omega C}$$

$$Z_1 Z_2 = j\omega L \times \frac{1}{j\omega C}$$

$$R_o^2 = \frac{L}{C}$$

$$\boxed{R_o = \sqrt{\frac{L}{C}}}$$

$$X_L = 2\pi f L$$

$$X_L \propto f$$

$$X_C = \frac{1}{2\pi f C}$$

Cut off frequency – (f_c)

$$Z_{oT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

$$Z_1 = j\omega L \quad \text{and} \quad Z_2 = \frac{1}{j\omega C}$$

$$Z_{oT} = \sqrt{\frac{j\omega L}{j\omega C}\left(1 + \frac{\frac{j\omega L}{4}}{\frac{1}{j\omega C}}\right)} = \sqrt{\frac{L}{C}\left(1 + \frac{j^2\omega^2 LC}{4}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)}$$

Z_{0T} is real in the pass band, whereas $\omega = \omega_c$

$$\frac{\omega^2 LC}{4} < 1 \quad , \text{ Pass band range, where } Z_0 \text{ is real.}$$

$$\frac{\omega^2 LC}{4} > 1 \quad , \text{ Stop band range, where } Z_0 \text{ is imaginary.}$$

$$\frac{\omega^2 LC}{4} = 1 \quad \text{change over from pass band to Stop band.}$$

$\therefore$ at $\omega = \omega_c$

$$\frac{\omega_c^2 LC}{4} = 1$$

$$\omega_c^2 = \frac{4}{LC} \qquad (2\pi f_c)^2 = \frac{4}{LC}$$

$$4\pi^2 f_c^2 = \frac{4}{LC}$$

$$\boxed{f_c = \frac{1}{\pi\sqrt{LC}}}$$

Design of LPF -

$$R_0 = \sqrt{\frac{L}{C}} \qquad f_c = \frac{1}{\pi\sqrt{LC}} \quad \text{and } L = R_0^2 C$$

Putting value of L in $f_c = \dfrac{1}{\pi\sqrt{LC}}$

$$f_c = \frac{1}{\pi\sqrt{C}\sqrt{C}R_0} = \frac{1}{\pi C R_0}$$

$$\boxed{C = \frac{1}{\pi R_0 f_c}} \quad \text{——— (1)}$$

putting $L = R_0^2 C$ we get $L = \dfrac{R_0^2}{\pi R_0 f_c}$

$$\boxed{L = \frac{R_0}{\pi f_c}}$$

Chartersistic impedance $\quad Z_{0T} = R_0$ or $Z_{0\pi} = \infty$ when $\omega = 0$

$Z_{0T} = 0$ or $Z_{0\pi} = R_0$ when $\omega = \omega_c$

$$\boxed{\begin{array}{l} \text{Attenuation band} = \alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right) = \alpha \\ \text{Phase band} = \beta = 2\sin^{-1}\left(\frac{f}{f_c}\right) \end{array}}$$

Constant K High Pass Filter - (HPF) - A high pass filter is one which attenuates all the frequencies below a specified cutoff frequency f_c and passes all the above f_c frequencies.

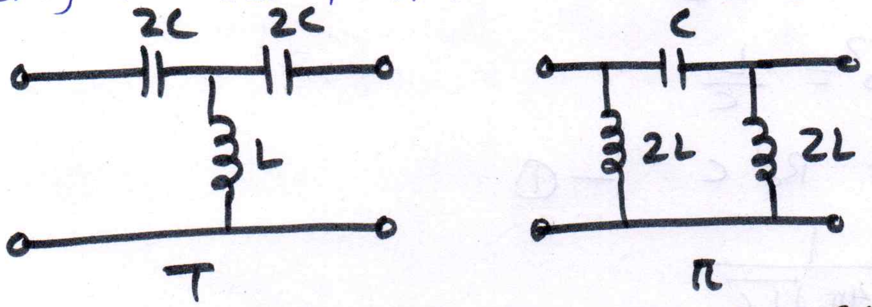


fig. - Prototype High Pass filter

$$Z_1 \cdot Z_2 = R_o^2 = \sqrt{\left(\frac{1}{j\omega C}\right) \times j\omega L}$$

$$\boxed{R_o = \sqrt{\frac{L}{C}}}$$

Cut off frequency -

$$Z_{OT} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

where $Z_1 = \frac{1}{j\omega C}$, $Z_2 = j\omega L$

$$Z_{OT} = \sqrt{\frac{j\omega L}{j\omega C}\left(1 + \frac{1}{4 j\omega C \times j\omega L}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

If $\frac{1}{4\omega^2 LC} < 1$, Z_{OT} is real, pass band

$\frac{1}{4\omega^2 LC} > 1$, Z_{OT} is imaginary, stop band.

$$\omega = \omega_c \quad . \quad \frac{1}{4\omega^2 LC} = 1$$

$$\omega^2 = \frac{1}{4LC}$$

$$(2\pi f_c)^2 = \frac{1}{4LC}$$

$$4\pi^2 f_c^2 = \frac{1}{4LC} \qquad \boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

Design of HPF -

$$R_0 = \sqrt{\frac{L}{C}}$$

$$R_0^2 = \frac{L}{C}$$

$$L = R_0^2 C \quad \text{— ①}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$f_c = \frac{1}{16\pi^2 LC}$$

$$f_c^2 = \frac{1}{16\pi^2 R_0^2 C.C} \qquad \text{Putting } L = R_0^2.C$$

$$= \frac{1}{16\pi^2 R_0^2 C^2}$$

$$C^2 = \frac{1}{16\pi^2 f_c^2 R_0^2}$$

$$\boxed{C = \frac{1}{4\pi f_c R_0}}$$

Putting value of C in equ^n ①

$$L = R_0^2 \times \frac{1}{4\pi f_c R_0} = \frac{R_0}{4\pi f_c}$$

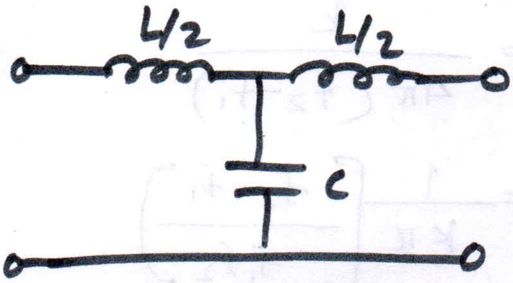
$$\boxed{L = \frac{R_0}{4\pi f_c}}$$

$$Z_{0T} = R_0\sqrt{1-\left(\frac{f_c}{f}\right)^2} \qquad Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

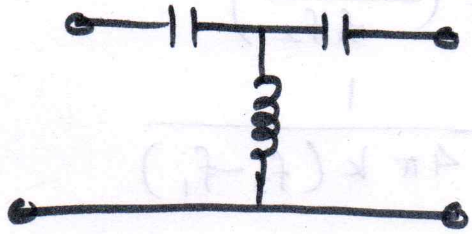
$$\boxed{\text{Attenuation Band} \quad \alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right)}$$

$$\boxed{\text{Phase constant} \quad \beta = 2\sin^{-1}\left(\frac{f_c}{f}\right)}$$

Constant K - Band Passfilter (BPF) -



L.P.F.



H.P.F.

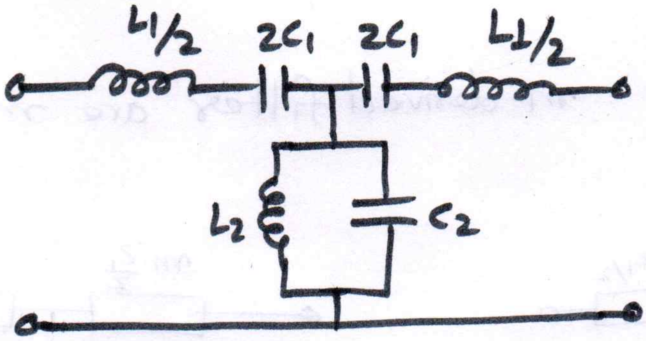


fig. - B.P.F.

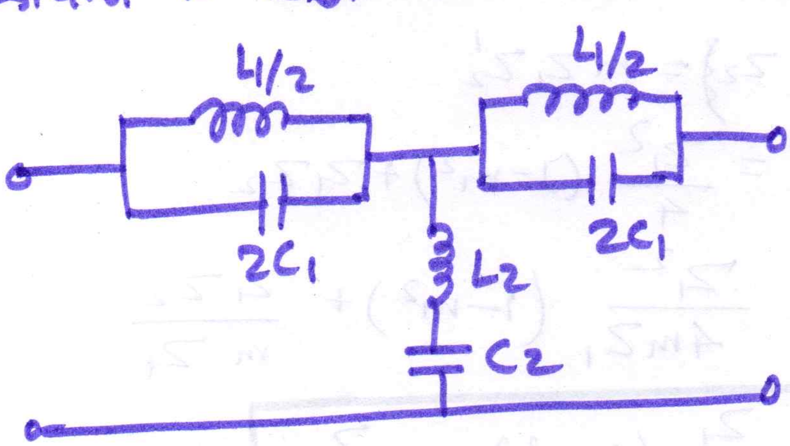
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

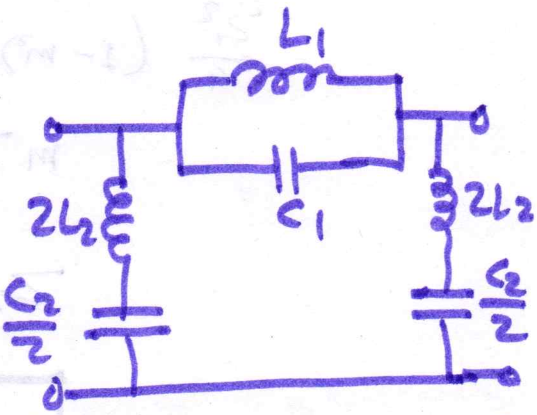
$$f_0 = \sqrt{f_1 f_2}$$

$$L_1 = \frac{k}{\pi(f_2 - f_1)} \qquad L_2 = \frac{(f_2 - f_1)k}{4\pi f_1 f_2}$$

$$C_1 = \frac{f_2 - f_1}{4\pi k f_1 f_2} \qquad C_2 = \frac{1}{k(f_2 - f_1)k}$$

Constant-K BSF



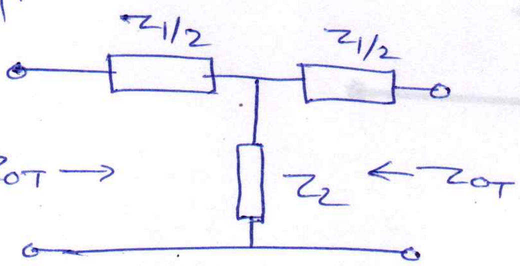


for BSF $f_0 = \sqrt{f_1 f_2}$

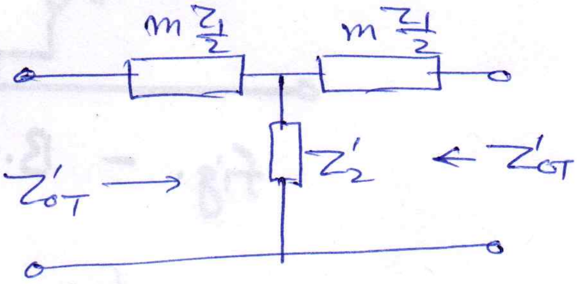
$$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right) \qquad L_2 = \frac{k}{4\pi(f_2 - f_1)}$$

$$C_1 = \frac{1}{4\pi k(f_2 - f_1)} \qquad C_2 = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right]$$

m-Derived filter - m-derived filter are modified form of prototype filter.



Constan K filter



m- derived filter

$$Z_{OT} = Z'_{OT}$$

$$\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{m^2 Z_1^2}{4} + m Z_1 Z_2'}$$

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + m Z_1 Z_2'$$

$$\frac{Z_1^2}{4}(1 - m^2) + Z_1 Z_2 = m Z_1 Z_2'$$

$$m Z_1 Z_2' = \frac{Z_1^2}{4}(1 - m^2) + Z_1 Z_2$$

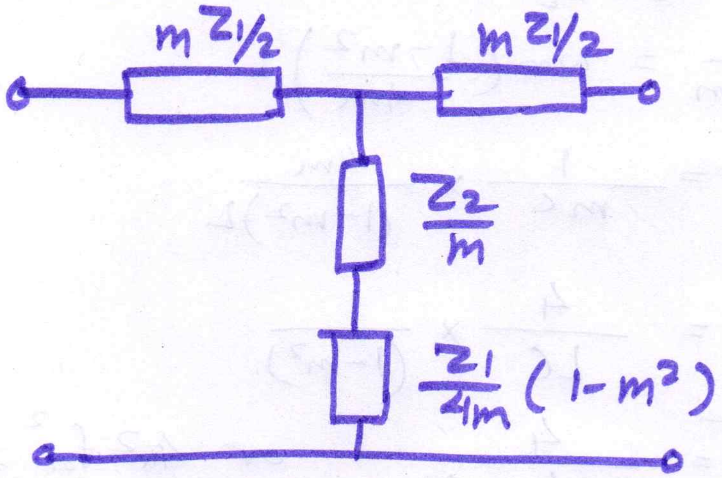
$$Z_2' = \frac{Z_1^2}{4 m Z_1}(1 - m^2) + \frac{Z_1 Z_2}{m Z_1}$$

$$\boxed{Z_2' = \frac{Z_1}{4m}(1 - m^2) + \frac{Z_2}{m}} \quad \text{—— (1)}$$

From equation ① $\frac{1-m^2}{4m}$ should be +ve to realise the impedances Z_2' $0 < m < 1$

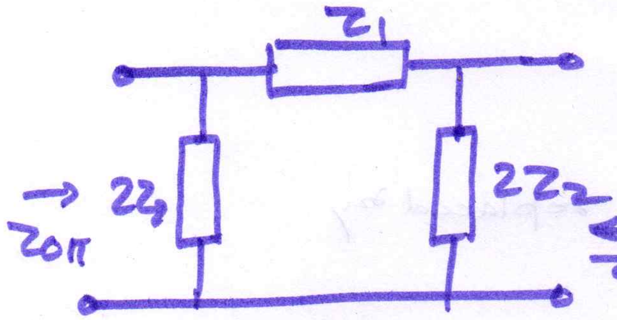
If $m=1$ circuit becomes prototype filter

So m is kept in between 0 and 1.

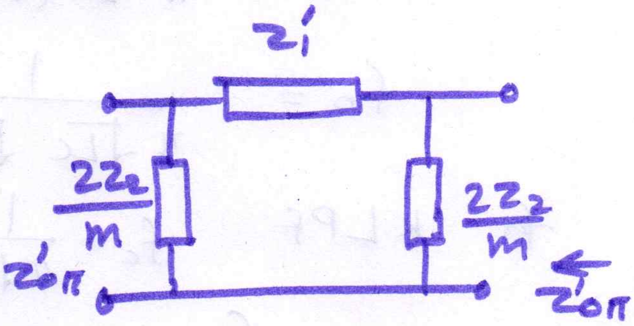


for a π filter, same method is adopted.

$$Z_{0\pi} = Z_{0\pi}'$$

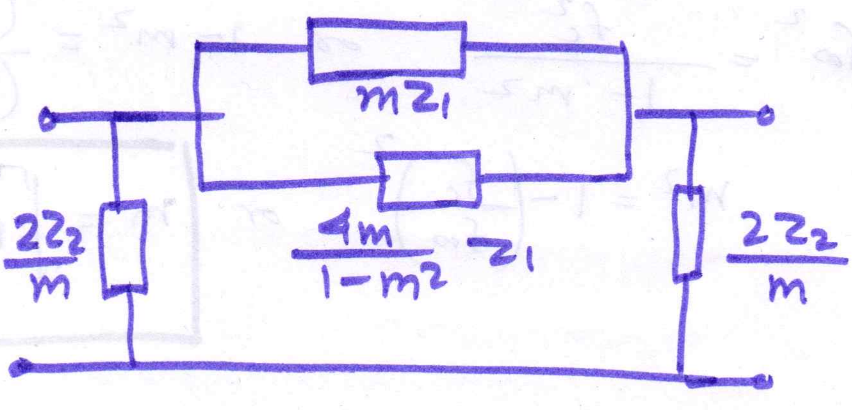


Constan k section



m- Derived section

$$Z_1' = \frac{(mZ_1)\left(\frac{4m}{1-m^2}\cdot Z_2\right)}{\frac{4m}{1-m^2}\cdot Z_2 + mZ_1}$$



m-derived LPF.

In m-derived filter

Series arm = mL Shunt arm = $mC + \frac{1-m^2}{4m}L$

frequency of infinite attenuation f_∞

$$X_C = X_L$$

$$\frac{1}{\omega_\infty C \cdot m} = \omega_\infty \left(\frac{1-m^2}{4m}\right) L$$

$$\omega_\infty^2 = \frac{1}{mC} \times \frac{4m}{(1-m^2)L}$$

$$= \frac{4}{LC} \times \frac{1}{(1-m^2)}$$

$$(2\pi f_\infty)^2 = \frac{4}{LC(1-m^2)} \quad \text{or} \quad 4\pi^2 f_\infty^2 = \frac{4}{LC(1-m^2)}$$

$$f_\infty^2 = \frac{1}{\pi^2 LC(1-m^2)}$$

$$f_\infty = \frac{1}{\pi\sqrt{LC}\sqrt{1-m^2}}$$

for K LPF $f_c = \frac{1}{\pi\sqrt{LC}}$ replaced by

$$f_\infty = \frac{f_c}{\sqrt{(1-m^2)}}$$

Determination of m

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

$$f_\infty^2 = \frac{f_c^2}{1-m^2} \quad \text{or} \quad 1-m^2 = \frac{(f_c)^2}{(f_\infty)^2}$$

$$m^2 = 1 - \left(\frac{f_c}{f_\infty}\right)^2 \quad \text{or} \quad \boxed{m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}}$$

Design of m-derived filter (LPF)

$$f_\infty = \frac{f_c}{\sqrt{(1-m^2)}} \quad \text{or} \quad m = \sqrt{1-\left(\frac{f_c}{f_\infty}\right)^2}$$

$$L = \frac{R_0}{\pi f_c} \quad \text{or} \quad C = \frac{1}{\pi f_c R_0}$$

m-derived HPF –

frequency of infinite $\quad X_L = X_C$

$$\omega_\infty \frac{L}{m} = \frac{1}{\omega_\infty \frac{4m}{1-m^2} C}$$

$$\omega_\infty^2 \text{ or } (2\pi f_\infty)^2 = \frac{1-m^2 \cdot m}{4m \cdot C \cdot L}$$

$$4\pi^2 f_\infty^2 = \frac{1-m^2}{4LC}$$

$$f_\infty^2 = \frac{1-m^2}{4LC\,4\pi^2} = \frac{1-m^2}{16\pi^2 LC}$$

$$f_\infty = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

for k HPF $f_c = \frac{1}{4\pi\sqrt{LC}}$ $\qquad f_\infty = f_c\sqrt{1-m^2}$

f_∞ is same for both T and π sections.

Determination of m.

$$f_\infty = f_c\sqrt{1-m^2}$$

$$f_\infty^2 = f_c^2(1-m^2)$$

$$1-m^2 = \frac{f_\infty^2}{f_c^2} \quad \text{or} \quad -m^2 = \frac{f_\infty^2}{f_c^2} - 1$$

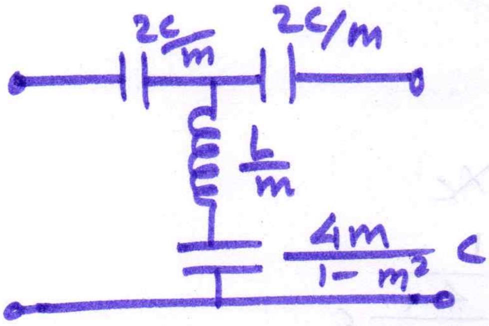
$$m^2 = 1 - \frac{f_\infty^2}{f_c^2} = 1 - \left(\frac{f_\infty}{f_c}\right)^2 \qquad \boxed{m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}}$$

Design of m-derived HPF

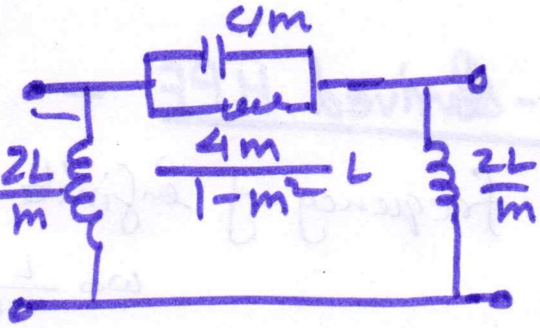
$$f_\infty = f_c\sqrt{1-m^2} \qquad m = \sqrt{1-\left(\frac{f_\infty}{f_c}\right)^2}$$

whereas the values of L and C are same as of constant K HPF.

$$L = \frac{R_0}{4\pi f_c} \quad \text{and} \quad C = \frac{1}{4\pi f_c R_0}$$

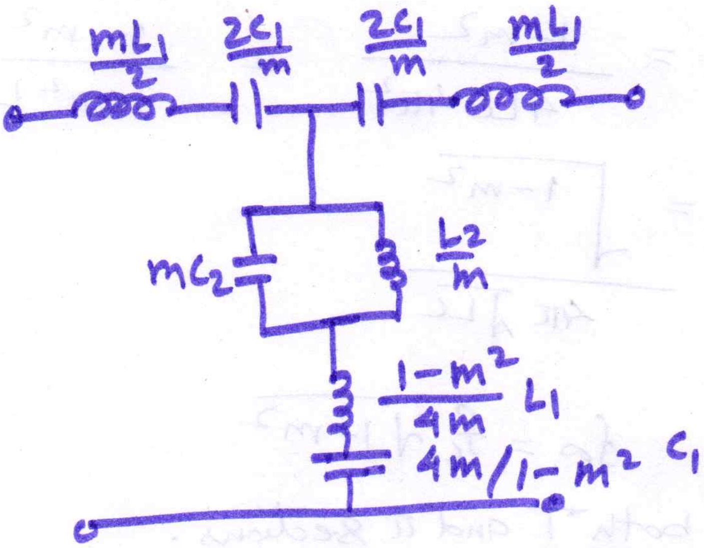


m-derived HPF



m-derived HPF

* m-Derived Band Pass Filter -



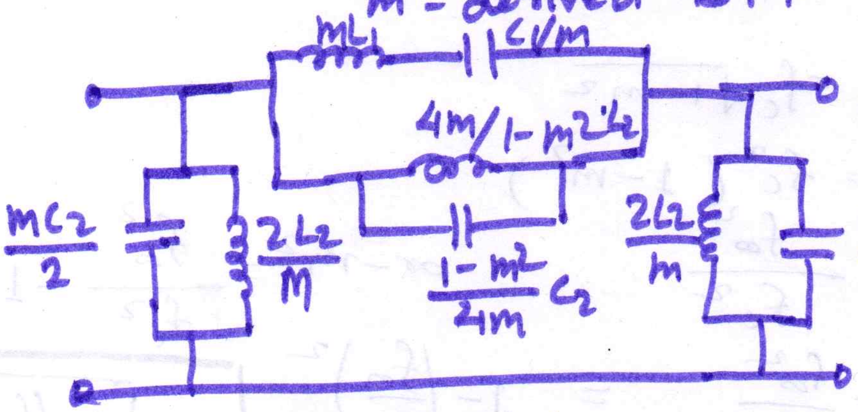
m-derived BPF T-section

$$L_1 = \frac{R_0}{\pi(f_2-f_1)}$$

$$C_1 = \frac{f_2-f_1}{4\pi f_1 f_2 R_0}$$

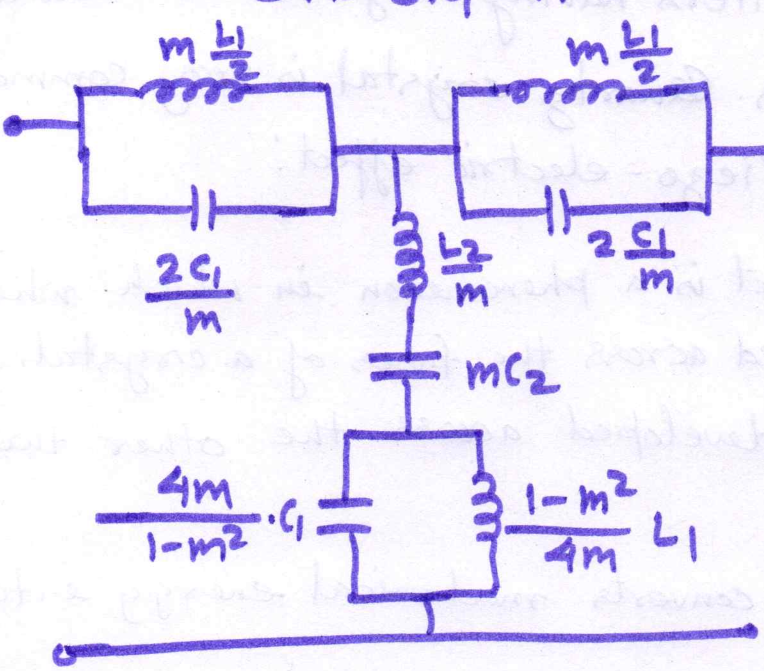
$$L_2 = \frac{(f_2-f_1)R_0}{4\pi f_1 f_2}$$

$$C_2 = \frac{1}{\pi R_0(f_2-f_1)}$$



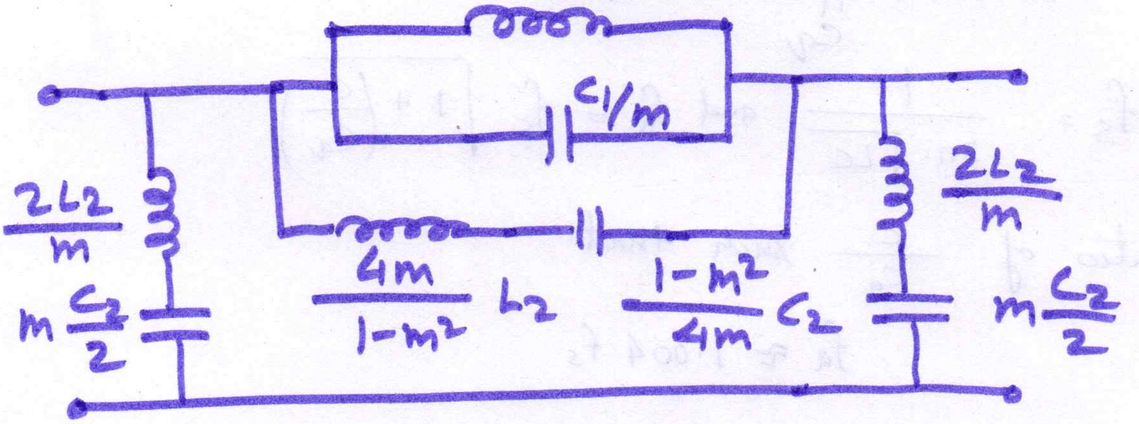
m-derived BPF π-section

* m-derived Band Stop Filter



$$m = \sqrt{1 - \left(\frac{f_{\infty 2} - f_{\infty 1}}{f_2 - f_1}\right)^2}$$

m-Derived BSF T-Section



m-derived BSF π Section

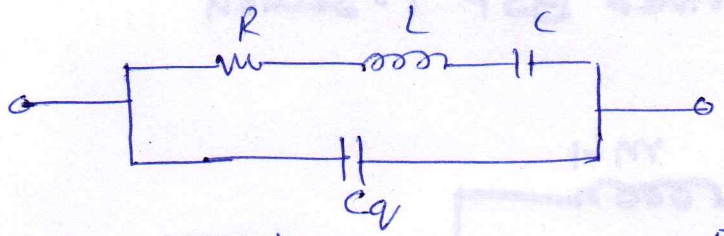
$$L_1 = \frac{R_0(f_2 - f_1)}{\pi f_1 f_2} \qquad C_1 = \frac{1}{4\pi R_0 (f_2 - f_1)}$$

$$L_2 = \frac{R_0}{4\pi (f_2 - f_1)} \qquad C_2 = \frac{(f_2 - f_1)}{\pi R_0 f_1 f_2}$$

* **Crystal filter** – Filters having crystals in their arms are called crystal filters. Quartz crystal is very commonly used in these filters. 'Piezo-electric effect'.

*. The Piezo-electric effect is a phenomenon in which when a mechanical force is applied across the faces of a crystal, it causes an emf to be developed across the other two surfaces of the crystal.

* Piezo-electric effect converts mechanical energy into electrical energy.



$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad f_a = f_s\sqrt{1 + \left(\frac{C}{C_q}\right)}$$

The ratio of $\frac{C}{C_q}$ such that

$$f_a \approx 1.004\, f_s$$

Design of crystal filter

let x is length y is breadth or z is height.

$$L = 115\frac{xy}{z}\ H \qquad C = 0.0032\frac{yz}{x}\ pf \qquad C_q = 0.4\frac{yz}{x}\ pf.$$

* Active Filter

Active filters are inductorless network consisting of lumped resistors and capacitors inter-connected with operational amplifiers.

* Advantages of active filter.
  * Compact size
  * Easier to tune
  * No loading effect
  * Economical
  * lesser Attenuation

* Type of active filter - Active filters also used as LPF, HPF BPF, BSF. All these filters use active as well as passive components. IC 741 is used as operational amplifier. LM 318 is also used.

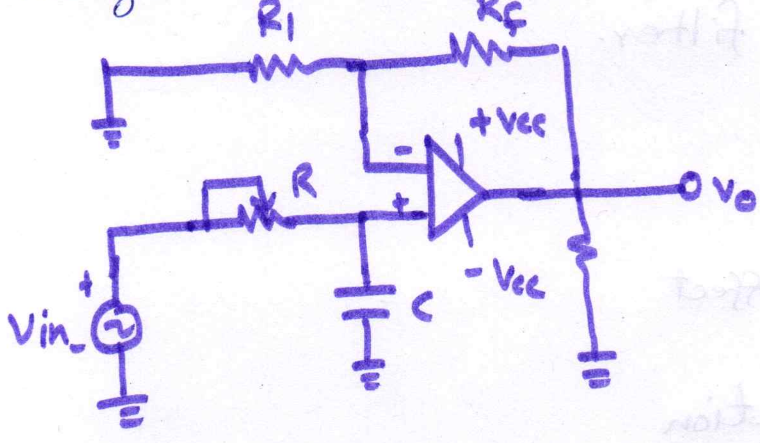
Type of different active filters.

Butterworth, Chebyshev and Causer filters are most commonly used filters.

* Butterworth filter has flat pass band as well as flat stop band. thats way often called Flat - flat active filter.
* The Chebyshev has ripple pass band & flat Stop band.
* Causer filter has ripple pass band as well as ripple stop band.

1st order Low Pass Butterworth filter -

Fig. shown first order LP filter having RC n/w ~~with~~ The OP-Amp is used in the non-inverting configuration. The resistors R_1 & R_f determine the gain of the filter.



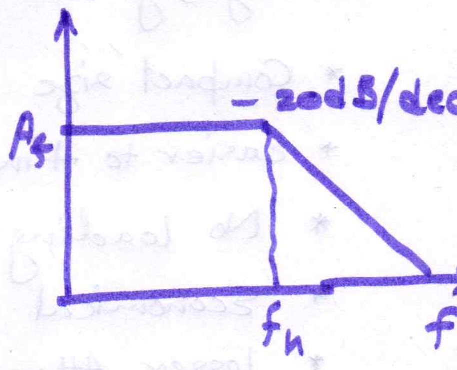


Fig. - Butterworth filter

$$\boxed{\frac{V_o}{V_{in}} = \frac{A_f}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}}$$

phase angle $\phi = -\tan f/f_H$

$$f_H = \frac{1}{2\pi RC}$$ high cut off frequency of filter.

At f_H the gain is $0.707\ A_f$.

$3\,dB = (20\log 0.707)$

The cutoff frequency is also known as $-3\,dB$ frequency, break frequency or Corner frequency.

c

2^{nd} order Low Pass Butterworth filter -

The Stop band response having a 40 dB/decade roll off is obtained with second order LPF.

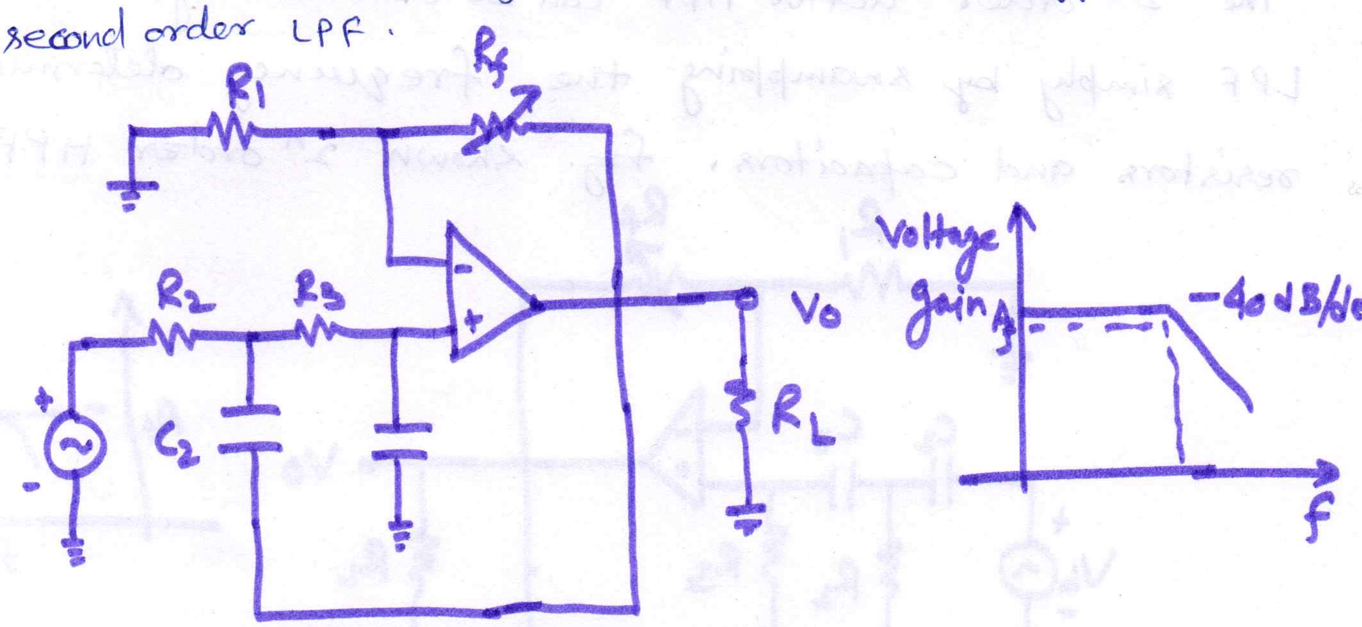


fig. 2nd order Butterworth LPF

$$\boxed{f_H = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{A_f}{\sqrt{1+(f/f_H)^4}}}$$

$A_f = 1 + \frac{R_f}{R_1}$ is the pass band gain of the filter.

1^{st} order HP Butterworth filter -

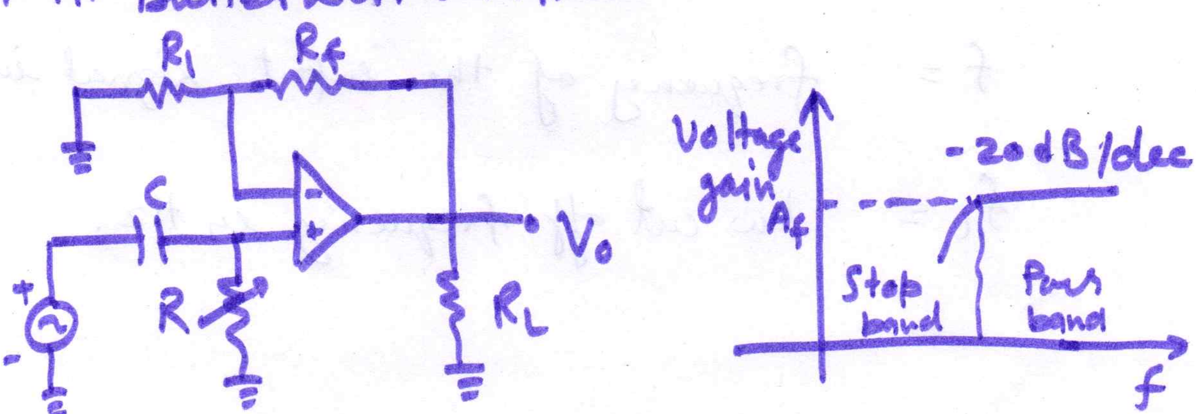


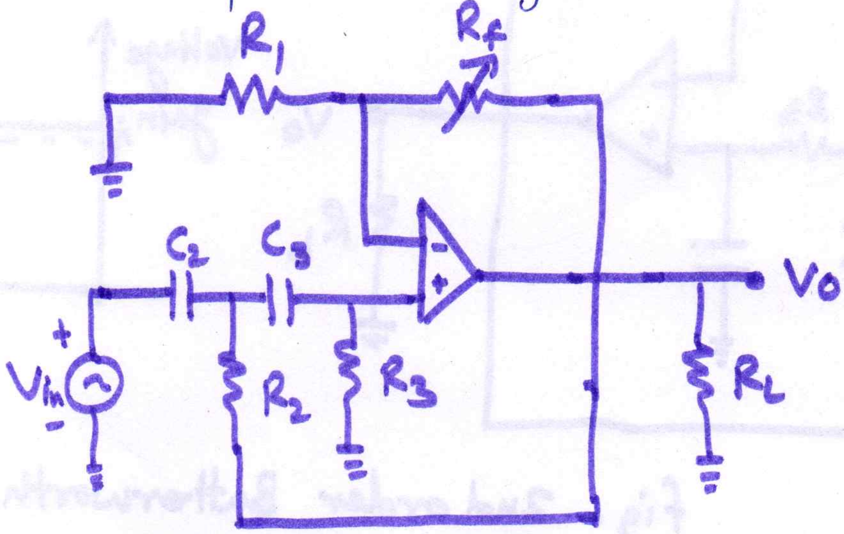
fig. 1^{st} order Butterworth HPF

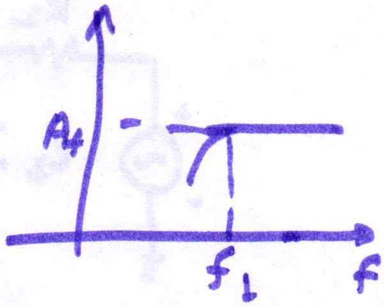
$$\boxed{f_L = \frac{1}{\pi RC}}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{A_f (f/f_L)}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}}}$$

* **2nd order HP Butter-worth filter -**

The 2nd order active HPF can be obtained from a 2nd order LPF simply by swampping the frequency determining resistors and capacitors. fig. shown 2n order HPF.





$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_f}{\sqrt{1 + \left(\frac{f_1}{f}\right)^4}}$$

Where A_f = 1.586, gain of the pass band ~~of~~ for the Second order Butterworth response

f = frequeny of the input signal in Hz.

f_1 = low cut off frequeny in Hz.

Transmission Lines

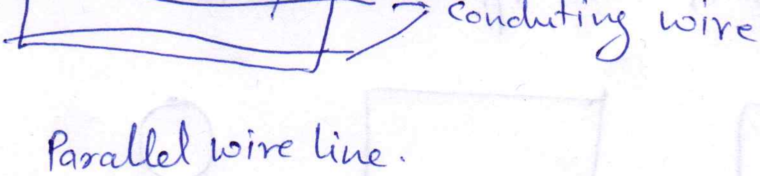
* Transmission line is a conductive medium consists of two or more conductors through which electrical energy is transmitted from one place to another. These lines are used b/w transmitter and receiver. Transmission line act as a medium or channel through which electric energy is sent from one place to another

* wires used for tube, fan, bulb or cable used for TV. wire connecting antenna with TV, phone lines etc.

* Transmission lines are classified depending upon their physical shape. Different forms of transmission lines are
  * Open line (parallel wire line)
  * Co-axial cable
  * wave guides
  * Optical fiber.

* Parallel wire -

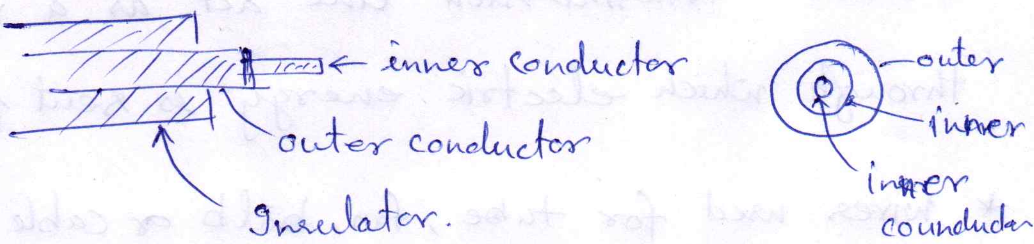
* This is most commonly used form of transmission line. This is considered as cheapest and simplest form. It consists two conductors seperated by a some distance and run parallel to each other. supported on insulator.



Parallel wire line.

* Resistance of wire in range of 300 Ω.

Co-axial Type - It consists of a central conductor surrounded by an outside conductor with dielectric b/w the inner and outer conductor.

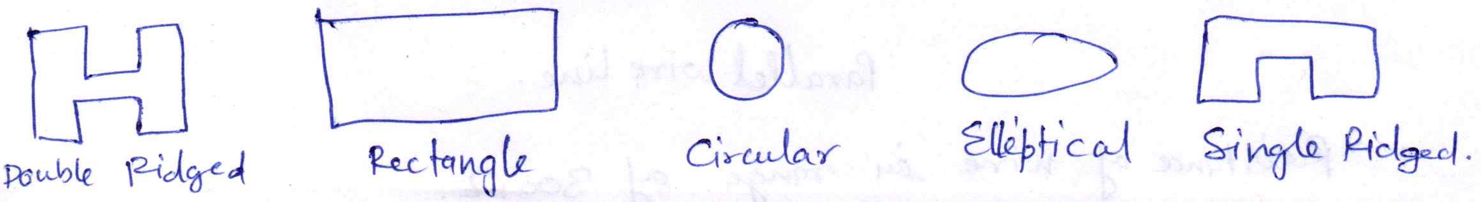


Co-axial conductor cable.

* Axis of both cables are same, that is why it known as co-axial cable.
* Impedance of these cables is 75 Ω. These are extensively used in frequency range extending upto 1 GHz.
* Above these frequencies co-axial cable is not suitable.

Wave Guide - It is a hollow conducting metallic tube of uniform cross section used for transmitting electromagnetic waves by successive reflection from the inner wall of the tube.

* Wave guide used above >3 GHz.
* Works on the total internal reflection principle.
* Different types & shape.



* **Optical fiber** - It consists of a very thin hollow glass fiber through which light wave is transmitted.

* Made it up of glass silica or plastic material.

* OFC are immune to both electromagnetic and electro static interferance.

* OFC are light in weight, high performance and high capacity transmission line.

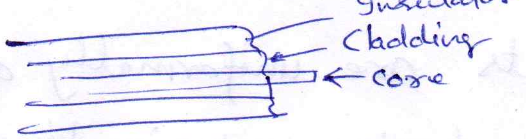
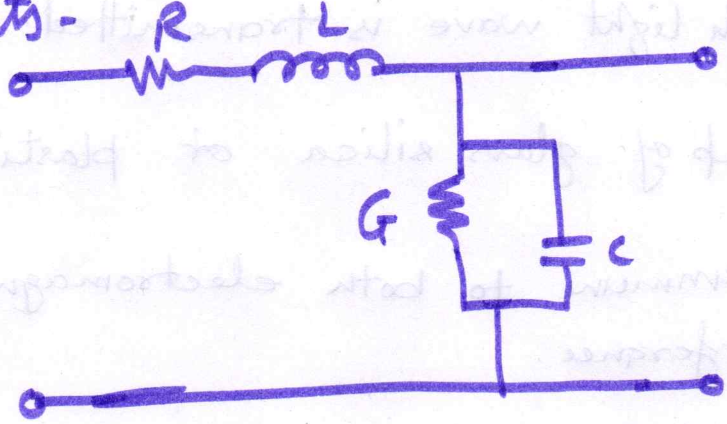


fig. OFC

* **Applications of Transmission line** -
  * Communication - TV, Radio, telephon, mobile. speech, fax. etc.
  * Circuits - Capacitor, resistor, inductor, filter. at high frequencies.
  * Impedance matching.
  * Home appliances & automation.

Equivalent Circuit of a transmission line.

* **Primary Constants** -



R = Resistance C = Capacitance L = Inductance

G = leakage conductance. R.

* These components are uniformally distributed along the entire length of transmission line.

* This is known as uniform transmission line or distributed parameter concept.

* These four parameters are called primary constants.

* Unit of these components are

L = loop inductance / unit length (H/km)
C = shunt capacitance / unit length (F/km)
R = loop resistance / unit length (Ω/km)
G = Shunt Conductance / Unit length (J/km)

* **Secondary Constants** -
  * Z_0 characteristic impedance
  * γ propagation constant.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{Z}{Y}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{Z \cdot Y}$$

Infinite Line – A line having infinite length.

* A signal fed into a line of infinite length will not be able to reach the far end (Load end) so conditions of load (open or short ckt) have not any effect at source end.

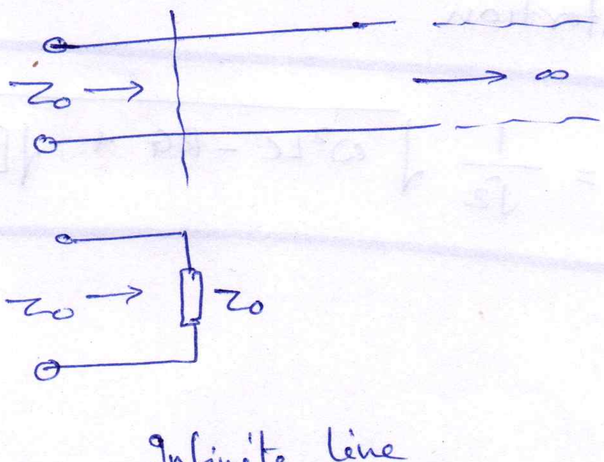
* The analysis of infinite line seperate the input and output conditions.

* Characteristics of infinite line -

(i) - Reflection will not occur from load end, Becacuse it is infinite length of line. Signal will not be able to reach far end.

(ii). The voltage & Current along the line will depends upon characteristic impedance Z_o and will not be affected by terminating impedance.

(iii) A short line terminated in Z_o characteristics impedance behaves as infinite length.



Infinite line

* **Distortion** — As the signal progresses on a line, it suffers. distortion. This is commonly known as line distortion. Actually signals transmitted over lines are normally complex because they consists of many frequency components.

* Causes of Distortion.

1)- The Z_0 of the line is related with frequency. As the signal consists of different frequencies. so terminating conditions will appear to be different for different frequencies.

2). The attenuation of the line also varies with frequency.

3)- The velocity of propagation also depends unpon frequency.

* Types of distortion. 1)- frequeny distortion
2)- Delay distortion.

frequeny Distortion.

$$\boxed{\alpha = \frac{1}{\sqrt{2}}\sqrt{RG - \omega^2 LC + \sqrt{R^2 + (\omega L)^2 (G^2 + \omega C^2)^2}}}$$

Delay Distortion

$$\boxed{\beta = \frac{1}{\sqrt{2}}\sqrt{\omega^2 LC - RG + \sqrt{[R^2 + (\omega L)^2] - [G^2 + (\omega C)^2]}}}$$

- **Distortion Less Line** - For ideal transmission, output signal should be exact replica of input signal i.e. signal should propagates to output terminal without any distortion.

$$\gamma = \sqrt{ZY}$$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{G}{C} + j\omega\right)}$$

$$= \sqrt{LC\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)} \quad \text{--- (I)}$$

from equ^n (I) $\quad \frac{R}{L} = \frac{G}{C}$

$$\gamma = \sqrt{LC}\left[\frac{R}{L} + j\omega\right]$$

$$\gamma = \frac{R}{L}\sqrt{LC} + j\omega\sqrt{LC} \quad \text{--- (II)}$$

$$\gamma = \sqrt{LC}\left[\frac{G}{C} + j\omega\right]$$

$$\gamma = \frac{G}{C}\sqrt{LC} + j\omega\sqrt{LC} \quad \text{--- (III)}$$

$$\gamma = \alpha + j\beta$$

Compare with equ^n (II) & (III)

$$\alpha = \frac{R}{L}\sqrt{LC} \qquad \frac{G}{C}\sqrt{LC}$$

$$\beta = \omega\sqrt{LC}$$

where α is independent of ω hence frequency.

β is related with ω hence frequency.

Velocity of propagation $\quad V_p = \frac{\omega}{\beta}$

$$V_p = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

As $\beta = \omega\sqrt{LC}$ and $\sqrt{LC}$ is constant means V_p is also independent of ω hence frequency.

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z_0 = \sqrt{\frac{L}{C}\left[\frac{\frac{R}{L}+j\omega}{\frac{G}{C}+j\omega}\right]}$$

$$\frac{R}{L} = \frac{G}{C}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

If the line is distortion free then characteristics impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

Also for a line to be distortion free condition is $\frac{R}{L} = \frac{G}{C}$

In a distortion free line, all frequency components have same amount of attenuation and velocity of propagation is also same for all frequency components means they are free from delay distortion. However output waveform's amplitude is less than input waveform but still have same shape.

* Methods to make line Distortion Free -

For a line to be distortion free $\frac{R}{L} = \frac{G}{C}$

But practically $\frac{R}{L} >> \frac{G}{C}$

So to make practically a distortion free line.

either $\frac{G}{C}$ is needed to be increased.

or $\frac{R}{L}$ is to be decreased.

* There are 3 methods to achive this conditions.

1. By Reducing R.

Resistance R can be decreased, which results in reduction of R/L.

* Due to this I^2R loss in line is reduced.
* But this method is costly and difficult to implement. Because to reduce resistance, a combination of parallel conductors is needed.
* Z_0 decrease due to which reflection increases.

2. By Decreasing C -

Capacitance C can be decreased, which results in increase G/c ratio. and hence.

* Attenuation α is decreased
* Z_0 increases, although not acceptable
* Installation cost also increases

Because C depends upon the construction of line it is not easy to reduce.

3. By increasing L -

Inductance L can be increased which results in reduction of R/L and this method

* is most commonly used.
* is called loading of line.
* is easy to implement.
* decrease α and hence distortion.
* improves Z_0 characteristic impedance, more importantly does not causes reflection
* requires additional amount of reactive power in transmiss line.

Change in G is practically not acceptable because increase in G increas in lekage current.

Reflection Coefficient (K) - The ratio of reflected voltage amplitude to the incident voltage amplitude.

for current - The ratio of reflected current to incident current.

$$K = \frac{V_r}{V_i} \quad \text{or} \quad k = -\frac{I_r}{I_i}$$

V_r = reflected voltage I_R = Reflected current

V_i = incident voltage I_i = incident current.

* Voltage ratio is called Voltage refection Coefficient (VRC).
* Current ratio is called Current reflection Coefficient (CRC).
* k is -ve in case of CRC becaus reflected current suffers a 180° ϕ shift at load end.

* Reflection coefficient is a measure of voltage and current reflected from the line due to impedance mismatch.

K is given by
$$K = \frac{Z_0 - Z_R}{Z_0 + Z_R}$$

If a line is terminated in Z_0 then $k = 0$

$$Z_0 = Z_R$$

$$K = \frac{Z_0 - Z_0}{Z_0 + Z_0} = \frac{0}{2} = 0$$

This line is said to be Non-resonant and flat line.

If line is terminated in open or short ckted there is a complete reflection

$$\therefore |K| = 1$$

Standing Waves -

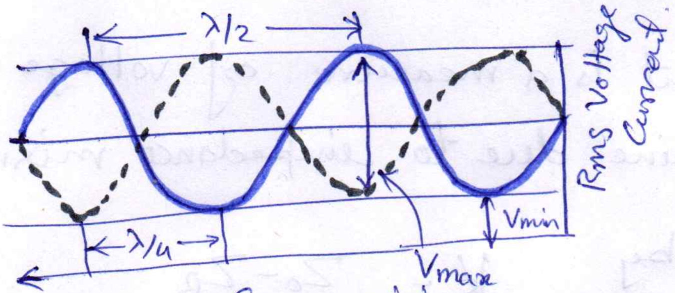
* The points where resultant signal i.e. voltage or current is maximum are known as voltage or current maxima.

* Where ~~resultant~~ resultant signal, voltage or current is minimum are known as voltage or current minima.

* At minima both the components are out of phase, while at minim both the components are in phase.

$$|V_{max}| = |V_i| + |V_r| \quad \text{and} \quad |V_{min}| = |V_i| - |V_r|$$

$$|I_{max}| = |I_i| + |I_r| \quad \text{and} \quad |I_{min}| = |I_i| - |I_r|$$



* **Standing wave Ratio (SWR) S** – It is the ratio of maximum and minimum magnitude of voltage or current.

$$VSWR = S = \frac{|V_{max}|}{|V_{min}|}$$

$$CSWR = S = \frac{|I_{max}|}{|I_{min}|}$$

$$VSWR = \frac{1+|K|}{1-|K|}$$

$$CSWR = \frac{1-|K|}{1+|K|}$$

Relation b/w S & K

$$|V_{max}| = |V_i| + |V_r|$$

$$|V_{min}| = |V_i| - |V_r|$$

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|V_i| + |V_r|}{|V_i| - |V_r|} = \frac{1 + |V_r/V_i|}{1 - |V_r/V_i|}$$

$$VSWR = \frac{1+|K|}{1-|K|}$$

$$S = \frac{1+|K|}{1-|K|}$$

$$S(1-|K|) = 1+|K|$$

$$S - S|K| = 1+|K|$$

$$S-1 = |K| + S|K| = |K|(1+S)$$

$$\boxed{|K| = \frac{S-1}{S+1}}$$

* Some facts about K & S.
    * Values of K lies between 0 and 1

$$0 < K < 1$$

K is the ratio of reflected to incident voltage amplitude.

$$K = V_r/V_i$$

Therefore reflected voltage can't be greater than incident voltage.

* In case of open and short circuit there is complete reflection. This means that reflected and incident voltages both are same.

$$V_r = V_i \qquad \therefore K = 1$$

For othe values of load impedance K is less than 1.

$$S = \frac{1+|k|}{1-|k|}$$

When $K = 0$, $S = \dfrac{1+0}{1-0} = 1$ (no reflection)

When $k = 1$, $S = \dfrac{1+1}{1-1} = \infty$ (complete reflection)

$$\boxed{1 < S < \infty}$$

Stubs – A short section of line can be used as impedance matching element. This short section of line is called stub. Stub is connected in shunt with main line at some suitable distance from load end. Stubs are used with open circuited or short circuited terminations.

* Short circuited stubs are mainly used for impedance matching. input impedance or short circuited and open circuited line is a pure reactance.

* Nature of this reactance whether capacitive or inductive depends upon the length of line used.

* A stub with short circuited load offer an inductive reactance if length is less than $\lambda/4$ and capacitive reactance, if length in b/w $\lambda/4$ and $\lambda/2$.

* Advantages of Stub –
  1)- The Z_0 of transmission line remains constant.
  2)- As the stubs are connected in shunt therefor length of line remains same.
  3)- A stub is used for a fixed frequency but it can be adjusted for different frequⁿ Ranges.
  4)- This method is economical.

* Types of Stub – Two types ①. Single ② Double Stub Matching