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Indefinite Integral - (अनिश्चित समाकलन)

सूत्र:

(i) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ $\qquad$ $\int 1\,dx = x + c$

(ii) $\int \cos x\,dx = \sin x + c$

(iii) $\int \sin x\,dx = -\cos x + c$

(iv) $\int \sec^2 x\,dx = \tan x + c$

(v). $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

(vi) $\int \sec x \tan x\,dx = \sec x + c$

(vii) $\int \operatorname{cosec} x \cdot \cot x\,dx = -\operatorname{cosec} x + c$

(viii) $\int \frac{1}{x}\,dx = \log|x| + c$

(ix) $\int e^x\,dx = e^x + c$

(x) $\int e^{-x}\,dx = -e^{-x} + c$

Questions

(I) (i) $\int \sin 2x \, dx = \dfrac{-\cos 2x}{2} + C$

(II) $\int 2 \sin 3x \, dx = -\dfrac{2\cos 3x}{3} + C$

(III) $\int \frac{1}{3} \cos 4x \, dx = \dfrac{\sin 4x}{12} + C$

(V) $\int 8 \sec^2 8x \, dx = \dfrac{\cancel{8} \tan 8x}{\cancel{8}} + C$

$= \tan 8x + C$

(VI) $\int \operatorname{cosec}^2 2x \, dx = -\dfrac{\cot 2x}{2} + C$

(VII) $\int \sec 5x \cdot \tan 5x \, dx = \dfrac{\sec 5x}{5} + C$

(2) $\int \cos(5 - 3x) \, dx$

$= \dfrac{\sin(5-3x)}{(-3)} + C$

$= -\dfrac{\sin(5-3x)}{3} + C$

(ii) $\int \sin\left(\frac{3x}{4}+5\right) dx$

$$= \frac{-\cos\left(\frac{3x}{4}+5\right)}{3/4} + C$$

$$= \frac{-4}{3}\cos\left(\frac{3x}{4}+5\right) + C$$

(3) $\int \sin^2 x \, dx.$

we know that

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$$= \int \frac{(1-\cos 2x)}{2} dx$$

$$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + C$$

(iv) $\int \cos^2 x \quad dx.$

we know that

$\therefore \cos 2x = 2\cos^2 x - 1$

$\frac{\cos 2x + 1}{2} = \cos^2 x$

$$= \int \left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$$

(iii) $\int \sin^3 x \, dx$ =

we know that

$\sin 3x = 3\sin x - 4\sin^3 x$

$4\sin^3 x = 3\sin x - \sin 3x$

$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$= \int \left(\frac{3\sin x - \sin 3x}{4}\right) dx$

$= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + C$

(iv) $\int \cos^3 x \, dx$.

we know that

$\cos 3x = 4\cos^3 x - 3\cos x$

$\frac{\cos 3x + 3\cos x}{4} = \cos^3 x$

$= \int \frac{(\cos 3x + 3\cos x)}{4} dx$

$= \frac{1}{4}\left(\frac{\sin 3x}{3} + 3\sin x\right) + C$

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(v) $\int 3\,cosec^2 x + 2\sin 3x \; dx$

$= -3\cot x - \frac{2}{3}\cos 3x + C$

(vi) $\int \sin x . \sec^2 x \; dx$

$= \int \sin x \times \frac{1}{\cos^2 x} dx = \int \frac{\sin x}{\cos x} \times \frac{1}{\cos x} dx$

$= \int \tan x . \sec x \, dx = \sec x + C$ Ans

(vii) $\int \frac{\sec x}{\sec x + \tan x} dx$

$= \int \frac{\sec x}{\sec x + \tan x} \times \frac{(\sec x - \tan x)}{(\sec x - \tan x)} dx$

$= \int \frac{\sec x(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$

we know that
$\sec^2 x - \tan^2 x = 1$

$= \int \sec x(\sec x - \tan x) dx$

$= \int (\sec^2 x - \sec x . \tan x)\, dx$

~~we~~ know that

$\int \sec^2 x\, dx = \tan x$

$\int \sec x . \tan x = \sec x$.

$= (\tan x - \sec x + C)$ Ans

Ques $\int (4e^{3x} + 1)\, dx = \frac{4e^{3x}}{3} + x + C$

$= \left(\frac{4}{3} e^{3x} + x + C\right)$ Ans

Ques $\int (ax^2 + bx + c)\, dx$

$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C'$ Ans

Ques $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = \int \left(x^{1/2} + x^{-1/2}\right) dx$.

$= \frac{x^{1/2+1}}{1/2+1} + \frac{x^{-1/2+1}}{-1/2+1} + C$

$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3} x^{3/2} + \frac{2}{1} x^{1/2} + C$ Ans

* समाकलन की विधियाँ

(i) प्रतिस्थापन द्वारा समाकलन
(ii) आंशिक भिन्नो द्वारा समाकलन
(iii) खण्डशः समाकलन

(*) प्रतिस्थापन विधि द्वारा समाकलन

Ques $\int \sin mx \, dx =$

let $mx = t$

$m\,dx = dt \Rightarrow dx = \frac{dt}{m}$

$= \int \sin t \frac{dt}{m}$

$= \frac{1}{m} \int \sin t \, dt = \frac{1}{m}(-\cos t)$

$= -\frac{1}{m} \times \cos mx$

$= -\frac{\cos mx}{m} + C$ Ans

Que $\int \sin 2x \, dx$.

Ans $\int \sin 2x \, dx$.

let $2x = t$

$2\,dx = dt$

$dx = \frac{dt}{2}$

$= \int \sin t \times \frac{dt}{2}$

$= \frac{1}{2}\int \sin t \, dt = -\frac{\cos t}{2} + C$

$= -\frac{\cos 2x}{2} + C$ Ans

Ques $\int \sin^3 x \cos^2 x \, dx$.

Solu^n $\int \sin^3 x \cos^2 x \, dx$

$= \int \sin x \cdot \sin^2 x \cdot \cos^2 x \, dx$

$= \int \sin x \, (1-\cos^2 x) \cos^2 x \, dx$

$= \int \sin x \, (\cos^2 x - \cos^4 x) \, dx$

let $\cos x = t$

diff. w.r.t. x

$-\sin x \, dx = dt$

$dx = -\frac{dt}{\sin x}$

$$= -\int \sin x \, (t^2 - t^4) \times \frac{dt}{\sin x}$$

$$= -\int (t^2 - t^4)\, dt$$

$$= -\left(\frac{t^3}{3} - \frac{t^5}{5}\right) + C$$

$$= -\left(\frac{\cos^3 x}{3} - \frac{\cos^5 x}{5}\right) + C$$ Ans

Ques $\int \frac{dx}{1+\sin x}$

Solun $\int \frac{dx}{1+\sin x} = \int \frac{1}{(1+\sin x)} \times \frac{(1-\sin x)}{(1-\sin x)}\, dx$

$$= \int \left(\frac{1-\sin x}{(1-\sin^2 x)}\right) dx = \int \frac{1-\sin x}{\cos^2 x}\, dx$$

$$= \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int (\sec^2 x - \tan x . \sec x)\, dx$$

$$= \tan x - \sec x + C$$ Ans

Ques $\int \frac{dx}{1+\tan x}$

Solu $\int \frac{dx}{1+\sin x/\cos x} = \int \frac{\cos x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int \frac{2\cos x}{\cos x+\sin x} dx = \frac{1}{2}\int \frac{(\cos x+\sin x)+(\cos x-\sin x)}{(\cos x+\sin x)}$

$= \frac{1}{2}\int \cancel{\frac{\cos x+\sin x}{\cos x+\sin x}} + \frac{\cos x-\sin x}{\cos x+\sin x)} dx$

$= \frac{1}{2}\left[\int 1 dx + \int \frac{\cos x-\sin x}{\cos x+\sin x)} dx\right.$

let $\cos x+\sin x = t$

diff w.r.t x

$(-\sin x+\cos x) dx = dt$

$= \frac{1}{2}\left[x + \log(\cos x+\sin x)\right) + C$

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Ques $\int \frac{\sin x}{\sin(x+a)} dx$

Soln $\int \frac{\sin(x-a+a)}{\sin(x+a)} dx$

we know that

$\sin(A-B) = \sin A . \cos B - \cos A . \sin B$

$= \int \frac{\sin((x+a)-a)}{\sin(x+a)} dx$

$= \int \frac{\sin(x+a)\cos a - \cos(x+a)\sin a}{\sin(x+a)} dx$

$= \int \frac{\sin\cancel{(x+a)}\cos a}{\sin\cancel{(x+a)}} - \frac{\cos(x+a)\sin a}{\sin(x+a)} dx$

$= \int \cos a - \cot(x+a)\sin a \, dx$

$= x\cos a - \sin a \log \sin(x+a) + C$

Ans

Ques $\int x\sqrt{x+2}\,dx$

let $(x+2) = t^2 \Rightarrow (t^2-2) = x$

diff w.r.t x

$(1+0)\,dx = 2t\,.dt$

$dx = 2t\,dt$

$\Rightarrow \int (t^2-2) \times t \times 2t\,dt$

$= \int 2t^2(t^2-2)\,dt$

$= 2\int (t^4 - 2t^2)\,dt$

$= 2\left(\frac{t^5}{5} - \frac{2t^3}{3}\right) + C$

$= 2\left[\frac{(x+2)^{5/2}}{5} - \frac{2(x+2)^{3/2}}{3}\right] + C$ Ans

Ques $\int \sin 4x \cdot \sin 8x\,dx$

Solun $\int \sin 4x \cdot \sin 8x\,dx$

we know that

$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

$= \frac{1}{2}\int 2\sin 4x \cdot \sin 8x\,dx$

$= \frac{1}{2}\Big[\cos(4x-8x) - \cos(4x+8x)\,dx$

$$= \frac{1}{2}\int [\cos 4x - \cos 12x)\, dx$$

$$= \frac{1}{2} \times \left(\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right) + C$$

Formulae

कुछ विशिष्ट फलनो के समाकलन

(1) $$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$$

(2) $$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$$

(3) $$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + C$$

(4) $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$$

(5) $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$$

(6) $$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$$

Ques $\int \frac{3x^2}{x^6+1}\,dx$

Solu^n $\int \frac{3x^2}{(x^6+1)}\,dx$

$\int \frac{3x^2}{(x^3)^2+1}\,dx$

let $x^3 = t$
diff. w.r.t x
$3x^2\,dx = dt$
$dx = dt/3x^2$

$= \int \frac{\cancel{3x^2}}{t^2+1} \times \frac{dt}{\cancel{3x^2}}$

$\int \frac{dt}{1+t^2}$ ∵ we know that $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$= \frac{1}{1}\tan^{-1}\frac{t}{1} + C$

$= \tan^{-1} x^3 + C$ Ans

Ques $\int \frac{1}{\sqrt{9-25x^2}}\,dx$

Solu^n $\int \frac{1}{\sqrt{9-25x^2}}\,dx$ $\int \frac{1}{\sqrt{3^2-(5x)^2}}\,dx$

let $5x = t$
$5\,dx = dt$
$dx = \frac{dt}{5}$

$= \int \frac{1}{\sqrt{3^2-t^2}} \times \frac{dt}{5}$

$$= \frac{1}{5} \int \frac{1}{\sqrt{3^2 - t^2}} \, dt$$

we know that $= \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C$

$$= \frac{1}{5} \times \sin^{-1} \frac{t}{3} + C$$

$$= \frac{1}{5} \times \sin^{-1} \frac{5x}{3} + C \quad \underline{\text{Ans}}$$

आंशिक भिन्नो द्वारा समाकलन

Ques $\int \frac{x^2+1}{x^2-5x+6} dx$.

Solun $\int \frac{x^2+1}{x^2-5x+6} dx$.

∵ अंश की घात हर की घात के बराबर है तो हर से अंश को भाग करेंगे।

$$x^2-5x+6 \overline{) x^2+1 } \; 1$$
$$\underline{x^2-5x+6}$$
$$5x-5$$

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$$

$$= \int \left(1 + \frac{5x-5}{x^2-5x+6} \right) dx = \int \left(1 + \frac{5x-5}{(x-3)(x-2)} \right) dx$$

$$\frac{5x-5}{(x-3)(x-2)} = \frac{A}{(x-3)} + \frac{B}{(x-2)}$$

$$\frac{5x-5}{\cancel{(x-3)(x-2)}} = \frac{A(x-2) + B(x-3)}{\cancel{(x-3)(x-2)}}$$

$$(5x-5) = (A+B)x - 2A - 3B$$

$$A+B = 5$$
$$-2A-3B = -5$$
$$2A+3B = 5$$

$A = -5, \quad B = 10$

$$= \int 1 + \frac{(-5)}{(x-2)} + \frac{10}{(x-3)} \, dx$$

$$= \int 1 - \frac{5}{(x-2)} + \frac{10}{(x-3)} \, dx$$

$$= x - 5\log(x-2) + 10\log(x-3) + C$$ Ans

Ques $\int \frac{3x-2}{(x+1)^2(x+3)} \, dx$

Sol: $\int \frac{3x-2}{(x+1)^2(x+3)} \, dx$

$$\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

$$\frac{3x-2}{\cancel{(x+1)^2(x+3)}} = \frac{A(x+1)(x+3) + B(x+3) + C(x+1)^2}{\cancel{(x+1)^2(x+3)}}$$

$$(3x-2) = (x+3)A(x+1) + B(x+3) + C(x^2+1+2x)$$

$$(3x-2) = A(x^2+4x+3) + B(x+3) + C(x^2+1+2x)$$

$$(3x-2) = (A+C)x^2 + (4A+B+2C) + (3A+3B+C)$$

गुणांकों की तुलना करने पर

$A + C = 0$

$4A + B + 2C = 3$

$3A + 3B + C = -2$

solving the above eq^n s.

$A = \frac{11}{4}, \quad B = -5/2, \quad C = -\frac{11}{4}$

$$\int \frac{\frac{11}{4}}{(x+1)} + \frac{-5/2}{(x+1)^2} + \frac{(-11/4)}{(x+3)} \, dx$$

$$= \int \frac{11}{4} \times \frac{1}{(x+1)} - \frac{5}{2(x+1)^2} - \frac{11}{4(x+3)} \, dx$$

$$= \frac{11}{4} \log(x+1) + \frac{5}{2(x+1)} - \frac{11}{4} \log(x+3) + C$$

Ans

खण्डशः समाकलन विधि द्वारा

$$\int f(x)\,g(x)\,dx = f(x)\int g(x)\,dx - \int \frac{d}{dx}f(x)\int g(x)\,dx$$

Ques. $\int x\cos x\,dx = x\int \cos x\,dx - \int \frac{d}{dx}x\int \cos x\,dx$

$= x(\sin x) - \int 1.\sin x\,dx$

$= x\sin x + \cos x + C$ Ans

Ques $\int x\,e^x\,dx$

$= x\int e^x dx - \int \frac{d}{dx}x.\int e^x\,dx$

$= xe^x - \int 1.\,e^x\,dx$

$= xe^x - e^x + C$ Ans

ILATE

*ii $\int e^x\left[f(x) + f'(x)\right]dx$ के प्रकार का समाकलन $= e^x f(x) + C$

Ques $\int e^x(\sin x + \cos x)\,dx$

$= \int (\overset{II}{e^x}\,\overset{I}{\sin x} + e^x\cos x)\,dx$

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$$= \int e^x \sin x \, dx + \int e^x \cos x \, dx$$

$$= e^x \sin x - \cancel{\int e^x \cos x \, dx} + \cancel{\int e^x \cos x \, dx}$$

$$= e^x \sin x + C \text{ Ans}$$

Ques $\int e^x \sec x (1 + \tan x) \, dx$

Solu^n $\int (e^x \sec x + e^x \sec x \cdot \tan x) \, dx$

$$= \int \underset{\text{II}}{e^x} \underset{\text{I}}{\sec x} \, dx + \int e^x \sec x \cdot \tan x \, dx$$

$$= \sec x \int e^x dx - \int \frac{d}{dx} \sec x \int e^x dx + \int e^x \sec x \cdot \tan x \, dx$$

$$= \sec x \, e^x - \cancel{\int e^x \sec x \cdot \tan x \, dx} + \cancel{\int e^x \sec x \cdot \tan x \, dx}$$

$$= e^x \sec x + C \text{ Ans}$$

कुछ अन्य प्रकार के समाकलन

(I) $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2} x \sqrt{x^2+a^2} + \frac{a^2}{2} \log|x+\sqrt{x^2+a^2}| + c$

(II) $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2} x \sqrt{a^2-x^2} + \frac{a^2}{2} \cdot \sin^{-1}\frac{x}{a} + c$

(III) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \log|x+\sqrt{x^2-a^2}| + c$

Ques $\int \sqrt{x^2+2x+5}\,dx$

$= \int \sqrt{(x^2+2x+1+4)}\,dx = \int \sqrt{(x+1)^2+2^2}\,dx$

by formula -

$= \frac{1}{2}(x+1) \sqrt{(x+1)^2+2^2} + \frac{4}{2} \log|(x+1)+\sqrt{x^2+2x+5}|$

$= \frac{1}{2}(x+1)\sqrt{(x^2+2x+5)} + 2\log|(x+1)+\sqrt{x^2+2x+5}| + c$

Ans

Ques $\int \sqrt{4-x^2}\,dx = \int \sqrt{2^2-x^2}\,dx$

$= \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2} \sin^{-1}\frac{x}{2} + c$

$= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + c$ Ans

निश्चित समाकलन
(definite Integral)

$$\int_a^b f(x)\,dx = f(b) - f(a)$$

(i) $\int_a^b x\,dx = \left(\frac{x^2}{2}\right)_a^b = \left(\frac{b^2}{2} - \frac{a^2}{2}\right)$ Ans

(ii) $\int_2^3 x^2\,dx = \left(\frac{x^3}{3}\right)_2^3 = \frac{1}{3}\left[3^3 - 2^3\right)$

$= \frac{1}{3}\left[27 - 8\right) = \frac{19}{3}$ Ans

(iii) $\int_2^3 \frac{1}{x}\,dx = \left(\log x\right)_2^3$

$= \left(\log 3 - \log 2\right) = \log\left(\frac{3}{2}\right)$

(iv) $\int_0^{\pi/2} \cos^2 x\,dx$

$\because \cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$\frac{(\cos 2x + 1)}{2} = \cos^2 x$

$= \int_0^{\pi/2} \frac{(1 + \cos 2x)}{2}\,dx$

$$= \int_0^{\pi/2} \frac{1+\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (1+\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left[\pi/2 + \frac{\sin 2\times\pi/2}{2}\right] - \frac{1}{2}\left[0 + \frac{\sin 2\times 0}{2}\right]$$

$$= \frac{1}{2}\left[\pi/2 + \frac{0}{2}\right] \qquad \because \sin\pi = 0$$

$$= \frac{\pi}{4} \text{ Ans.}$$

Ques $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

Solu^n $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \left(\frac{1}{1}\tan^{-1}\frac{x}{1}\right)_1^{\sqrt{3}}$

$$= \left[\tan^{-1}\sqrt{3} - \tan^{-1} 1\right]$$

$$= \left(\tan^{-1}\tan\frac{\pi}{3} - \tan^{-1}\tan \pi/4\right)$$

$$= \pi/3 - \frac{\pi}{4}$$

$$\frac{4\pi - 3\pi}{12} = \frac{\pi}{12} \text{ Ans}$$

Properties of definite integral

(i) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$

(iii) $\int_a^b f(x)\,dx = \int_a^c f(x)\cdot dx + \int_c^b f(x)\,dx$

(iv) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(v) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$

(vi) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

(vii) $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx.$ यदि $f(2a-x) = f(x)$

$= 0,$ यदि $f(2a-x) = -f(x)$

(viii) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$

यदि f एक सम फलन है $\cdot f(-x) = f(x)$

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$$\int_{-a}^{a} f(x)\,dx = 0.$$

यदि f एकविषम फलन है

$$f(-x) = -f(x)$$

Ques $$\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx.$$

by Property.

Soln $$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx \quad \text{—} (1)$$

$$I = \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)}\,dx$$

$$I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}\,dx \quad (2)$$

$(1) + (2)$

$$2I = \int_0^{\pi/2} \frac{\cancel{\sin^4 x + \cos^4 x}}{\cancel{\sin^4 x + \cos^4 x}}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$= (x)_0^{\pi/2}$$

$$2I = (\pi/2 - 0)$$

$$2I = \pi/2$$

$$\boxed{I = \pi/4}$$ Ans

Ques $$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Solu by property

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad (I)$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad (II)$$

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(i) + (ii)

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$= (x)_0^{\pi/2}$$

$$2I = \pi/2$$

$$I = \pi/4$$

Ques $\int_0^{\pi/4} \log(1+\tan x)\,dx$

Soln $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$

by property

$I = \int_0^{\pi/4} \log(1+\tan(\pi/4 - x))\,dx$

$I = \int_0^{\pi/4} \log\left(1+\frac{(\tan \pi/4 - \tan x)}{1+\tan \pi/4 . \tan x}\right) dx$

$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right) dx$

$I = \int_0^{\pi/4} \log\left(\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right) dx$

$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx$

$I = \int_0^{\pi/4} \log 2 - \underbrace{\int_0^{\pi/4} \log(1+\tan x)\,dx}_{}$

$$I = \int_0^{\pi/4} \log 2 \, dx - I$$

$$2I = \int_0^{\pi/4} \log 2 \, dx$$

$$2I = \log 2 \int_0^{\pi/4} 1 . dx$$

$$2I = \pi/4 . \log 2$$

$$\boxed{I = \pi/8 . \log 2}$$ Ans

Que

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$$

Soln

$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \text{--- (1)}$$

by using property

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{a-(a-x)}} dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \text{--- (2)}$$

(1) + (2)

$$2I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} + \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\, dx$$

$$= \int_0^a \frac{\cancel{(\sqrt{x}+\sqrt{a-x})}}{\cancel{(\sqrt{x}+\sqrt{a-x})}}\, dx$$

$$2I = \int_0^a 1.\, dx$$

$$2I = (x)_0^a$$

$$2I = (a-0)$$

$$\boxed{I = \frac{a}{2}}$$ Ans

Ques $\int \frac{dx}{e^x+e^{-x}}$

Soln $\int \frac{dx}{e^{-x}(e^{2x}+1)} = \int \frac{e^x\, dx}{(e^{2x}+1)}$

let $e^x = t$

diff w.r.t t

$e^x = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{e^x}$

$$\int \frac{\cancel{e^x}}{(t^2+1)} \times \frac{dt}{\cancel{e^x}}$$

some other formulae.

(i) $\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}x + c$

(ii) $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1}x + c$

(iii) $\int \frac{dx}{1+x^2} = \tan^{-1}x + c$

(iv) $\int \frac{dx}{1+x^2} = -\cot^{-1}x + c$

(v) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c$

(vi) $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1}x + c$

(vii) $\int a^x\, dx = \frac{a^x}{\log a} + c$

Q. $\int \frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}\, dx$

Solution $\int \frac{x^3 \sin(\tan^{-1}x^4)\, dx}{1+x^8}$

Let $\tan^{-1}x^4 = t$

diff. w.r.t. x

$\frac{1}{(1+x^8)} \times 4x^3\, dx = dt$

$dx = \frac{(1+x^8)\, dt}{2}$

$$= \int \frac{\cancel{x^3} \sin(t)}{\cancel{1+x^8}} \times \frac{\cancel{(1+x^8)}}{4\cancel{x^3}} \times dt$$

$$= \frac{1}{4} \int \sin t \, dt$$

$$= -\frac{1}{4} \cos t + C$$

$$= \frac{-1}{4} \cos(\tan^{-1} x^4) + C$$

Ans

Application of Integrals.
समाकलनों के अनुप्रयोग।

* वक्र $y = f(x)$, x अक्ष एवं रेखाओं $x = a$, तथा $x = b$. $(b > a)$ से घिरे क्षेत्र के क्षेत्रफल का सूत्र

$$\text{क्षे०} = \int_a^b y\,dx$$

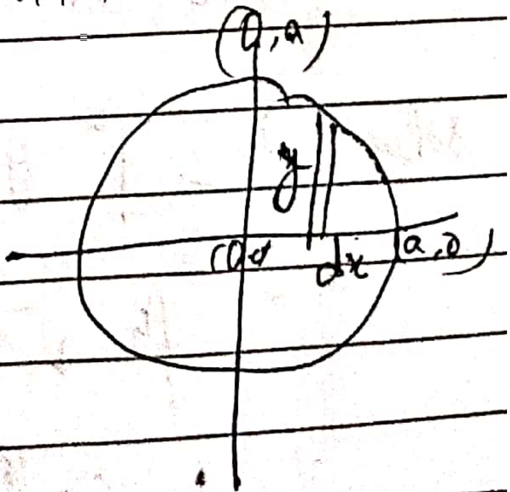
$$= \int_a^b f(x)\,dx$$

* वक्र $x = \phi(y)$, y-अक्ष एवं रेखाओं $y = c$, $y = d$ से घिरे क्षेत्र के क्षेत्रफल का सूत्र

$$\text{क्षे०} = \int_c^d x\,dy$$

$$= \int_c^d f(y)\,dy$$

Ques वृत्त $x^2 + y^2 = a^2$ का क्षे० ज्ञात कीजिए।



Solun

$$\text{क्षे०} = 4\int_0^a y\,dx$$

$$y^2 = a^2 - x^2$$

$$y = \sqrt{a^2 - x^2}$$

$$\text{क्षे०} = 4\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right] - 0.$$

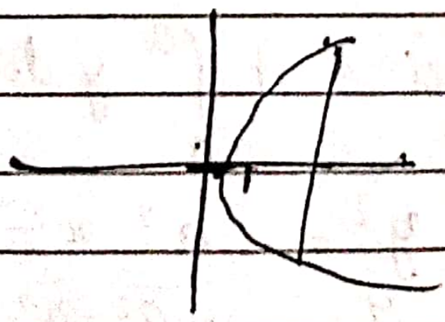
$$= 4\left[0 + \frac{a^2}{2} \times \frac{\pi}{2}\right.$$

$$= \frac{\cancel{4} \times \pi a^2}{\cancel{4}} = \pi a^2$$

$$\boxed{\text{वृत्त का क्षे०} = \pi a^2}$$

Ques वक्र $y^2 = x$ रेखाओं $x = 1$, $x = 4$. एवं x-अक्ष से घिरे क्षेत्र का क्षे०. ज्ञात कीजिए

solu क्षे० $= \int_1^4 y\,dx$



$$= \int_1^4 \sqrt{x}\,dx$$

$$= \left(\frac{x^{1/2+1}}{1/2+1}\right)_1^4$$

$$= \left(\frac{x^{3/2}}{3/2}\right)_1^4$$

$= \frac{2}{3}\left[x^{3/2}\right]_1^4 = \frac{2}{3}\left[4^{3/2} - 1\right]$

$= \frac{2}{3}\left[2^{2\times 3/2} - 1\right]$

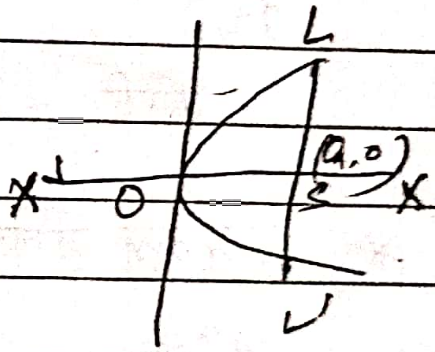
$= \frac{2}{3}\left[8 - 1\right] = \frac{2}{3}\times 7 = \frac{14}{3}$

क्षे० $= \frac{14}{3}$ Ans

Que. परवलय $y^2 = 4ax$ और उसके नाभिलम्ब से घिरे क्षेत्र का क्षे० ज्ञात करो।

Solu. क्षेत्र OLL'O का अभीष्ट क्षे०

= 2 (क्षे० OLSO का क्षे०)



$= 2\int_0^a y\,dx$

$= 2\int_0^a \sqrt{4ax}\,dx = 2\int_0^a 2\sqrt{a}\,\sqrt{x}\,dx$

$= 4\sqrt{a}\int_0^a x^{1/2}\,dx$

$= 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a = \frac{4\sqrt{a}\times a^{3/2}}{3/2}$

$= \frac{8}{3}a^{1/2}a^{3/2} = \frac{8}{3}a^2$ Ans